

Identification of a Phase-Field Model for Brittle Fracture in Transversely Isotropic Elastic Materials with Application to Spruce Wood Specimens under Compression

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Context and objectives

Context

- In the **furniture industry**, standardized tests are essential for certifying product safety and compliance.
- The **CEN** (*European Committee for Normalization*) provides standards to regulate product commercialization, such as **EN 747** for bunk and loft beds.
- Furniture is subjected to various **loads** and can be classified into three structural design types: **frame** (beam-type elements), **panel** (plate-type elements), and **combined** (both beam and plate elements).
- The wooden elements of these structures are assembled using a variety of **joining techniques**.

Motivation

- The **FCBA** (*Forêt Cellulose Bois-construction Ameublement*) institute conducts standardized validation tests for **furniture manufacturers**.
- These tests conducted under **static** or **dynamic** loads are essential for ensuring compliance with furniture commercialization standards.
- The **FCBA** wants to **numerically predict** the most frequent deterioration modes observed **experimentally**.



(a) Crossbeams cracking at bunk beds connections



(b) Transverse tear-out of particle board

Objective

- Reduce the number of **trips** between the **manufacturers** and the **FCBA** by numerically predicting the most frequent deterioration modes observed experimentally.
- **Numerically reproduce** simpler experimental tests on perforated **spruce specimens** subjected to **uniaxial compression**.
- Identify the **elastic** and **fracture** properties of wood-based materials from experimental measurements.

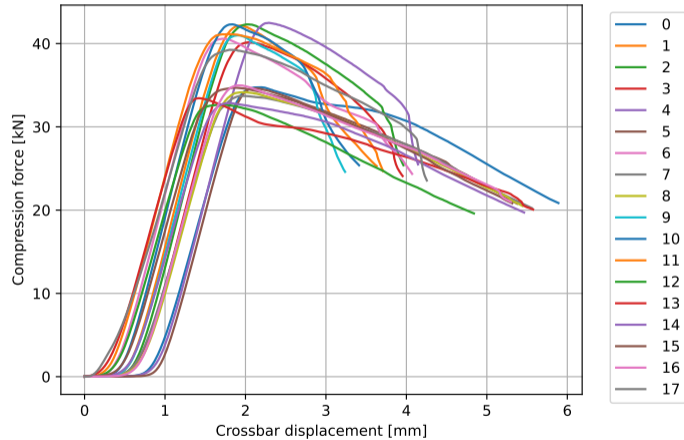
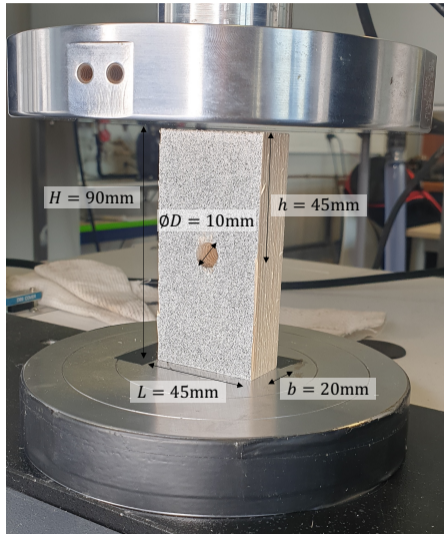
Assumptions

- The average elastic behavior of spruce wood is assumed to be **transversely isotropic** and **homogeneous**.
- The **phase-field** fracture method is suitable for simulating fracture mechanisms in **wood**.

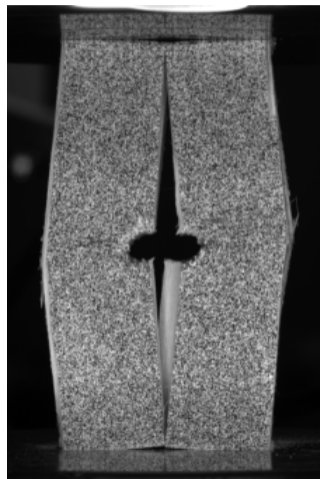
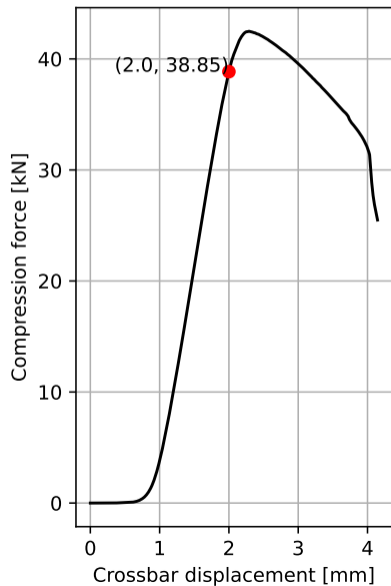
Outline

- 1 Context and objectives
- 2 Experimental compression tests
 - Observed cracking mechanisms
- 3 Identifying the elastic properties of wood
 - *Finite Element Model Updating*
 - Identified elastic properties
- 4 Identifying the damage properties of wood
 - Modeling brittle fracture using the Phase-Field method.
 - Phase-Field models for wood fracture
 - FEMU G_c
 - Identified critical energy release rate G_c
- 5 Conclusions

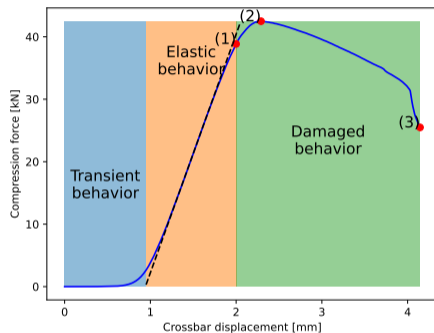
Experimental compression tests



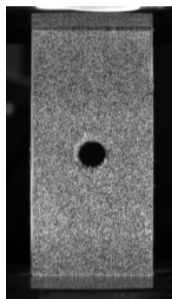
For each test, the images and force-displacement curves are **synchronized**.



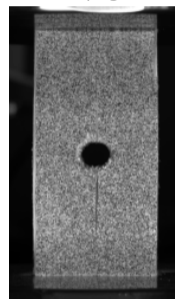
Observed cracking mechanisms



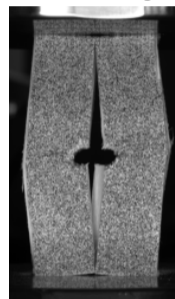
(1) Initiation



(2) Propagation

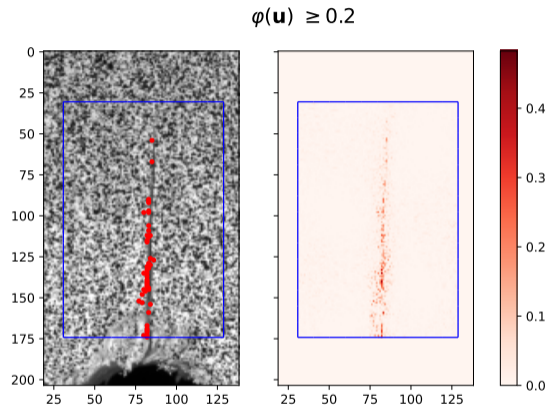
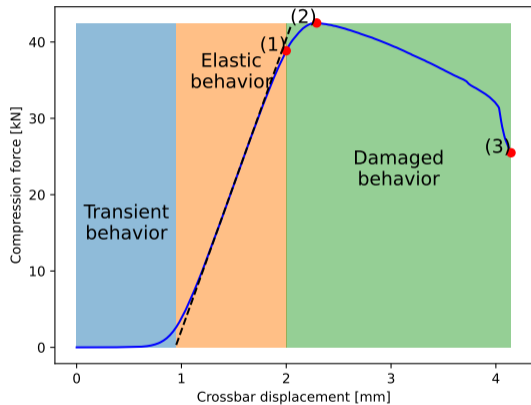


(3) Breaking



Typical force-displacement behavior observed during the tests

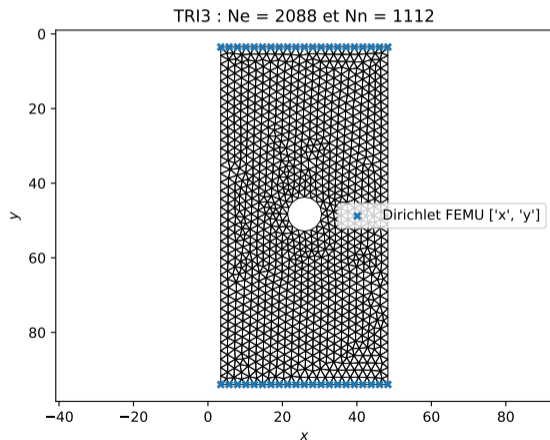
Cracking mechanisms

Identifying critical force through *Digital Image Correlation* (DIC)

$$\frac{[f(\mathbf{x}_p) - g(\mathbf{x}_p + \mathbf{u}(\mathbf{x}_p))]^2}{255^2} \geq 0.2$$

Identifying the elastic properties of wood

Finite Element Model Updating [Kavanagh and Clough 1971; Kavanagh 1972]

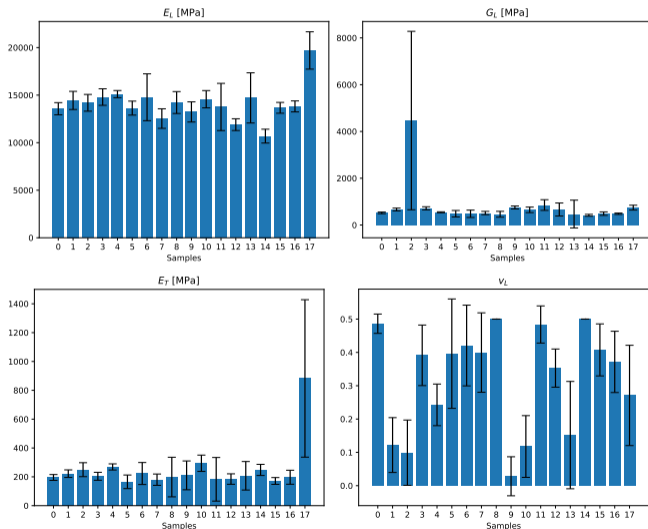


This method minimizes the least-squared difference between the experimentally measured displacement \mathbf{u}^{exp} and the numerically simulated displacement field $\mathbf{u}(\mathbf{p})$.

$$\min_{\mathbf{p}=(E_L, E_T, G_L, \nu_L) \in \mathcal{A}} \|\mathbf{u}(\mathbf{p}) - \mathbf{u}^{\text{exp}}\|^2$$

Here, $\mathcal{A} = \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times]0, 0.5[$ represents the admissible set for the parameter vector $\mathbf{p} = (E_L, E_T, G_L, \nu_L)$.

Identified elastic properties

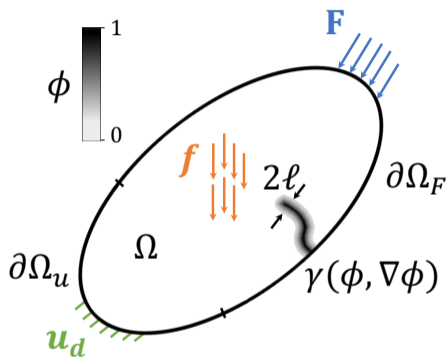


- Some dispersion occurs from one test to another, particularly in tests #2 and #17.
- It appears that the Poisson's ratio ν_L is not well identified with this kind of test

	mean	std	disp [%]
E_L [GPa]	13.78	1.21	8.76
E_T [MPa]	215.41	38.98	18.10
G_L [MPa]	541.49	89.07	16.45
ν_L	0.35	0.13	35.85

Elastic properties excluding samples with dispersion greater than 40%

Modeling brittle fracture using the Phase-Field method.



Regularized crack model
[Bourdin et al. 2000]

Method based on the minimization of the total energy functional.

$$\mathcal{E}(\mathbf{u}, \phi) = \Psi(\mathbf{u}, \phi) - \mathcal{P}(\mathbf{u}) + \Psi_c(\phi, \nabla\phi)$$

$$\Psi(\mathbf{u}, \phi) = \int_{\Omega} (g(\phi) \psi^+(\mathbf{u}) + \psi^-(\mathbf{u})) \, d\Omega$$

$$\mathcal{P}(\mathbf{u}) = \int_{\Omega} \mathbf{f} \cdot \mathbf{u} \, d\Omega + \int_{\partial\Omega_F} \mathbf{F} \cdot \mathbf{u} \, dS$$

$$\Psi_c(\phi, \nabla\phi) = \int_{\Omega} \frac{G_c}{c_w \ell} (w(\phi) + \ell^2 \|\nabla\phi\|^2) \, d\Omega$$

With the following constraints $\mathbf{u} = \mathbf{u}_D$ on $\partial\Omega_D$,
 $\dot{\phi} \geq 0$ and $\phi \in [0, 1]$.

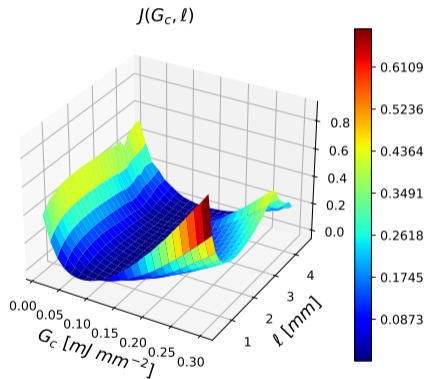
Despite numerous applications in various transversely or orthotropic elastic materials, the phase-field modeling approach for wood fracture simulation has received little attention.

Overview of existing wood fracture models

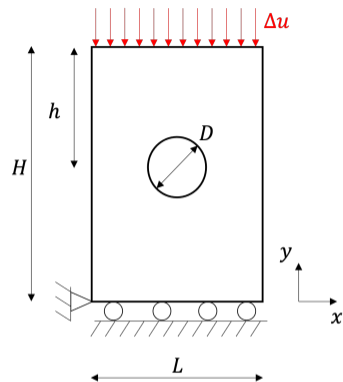
- Dynamic fracture models in wood fiber composites [Carlsson et al. 2018] and modeling of wood microstructure and intercellular fractures [Carlsson et al. 2020].
- Wood anisotropy and mode-dependent fracture models, with extension to orthotropic materials [Supriatna et al. 2022].
- Transversely isotropic or orthotropic elastic fracture models, focusing on asymmetric energy splits and potentially anisotropic fracture surface energies [Pech et al. 2022a; Pech et al. 2022b].

In this study, we use the model presented in [Nguyen et al. 2020], which employs a strain-based orthogonal split of the elastic strain energy density function to address asymmetric fracture ($\boldsymbol{\epsilon}^+ : \mathbb{C} : \boldsymbol{\epsilon}^- = \boldsymbol{\epsilon}^- : \mathbb{C} : \boldsymbol{\epsilon}^+ = 0$).

$$J(G_c, \ell) = \left(\frac{f(G_c, \ell) - f^{\text{exp}}}{f^{\text{exp}}} \right)^2$$



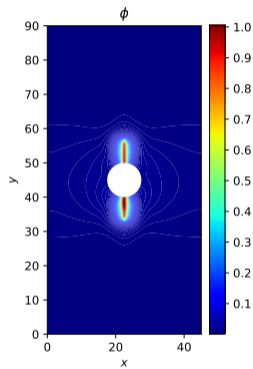
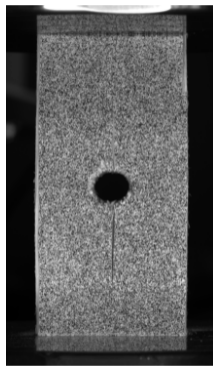
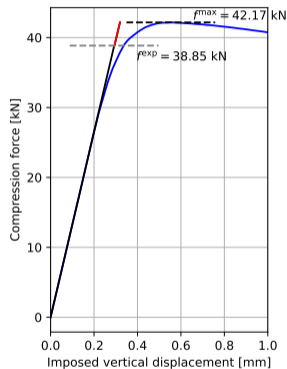
The characteristic length ℓ has a negligible influence compared to the critical energy release rate G_c .



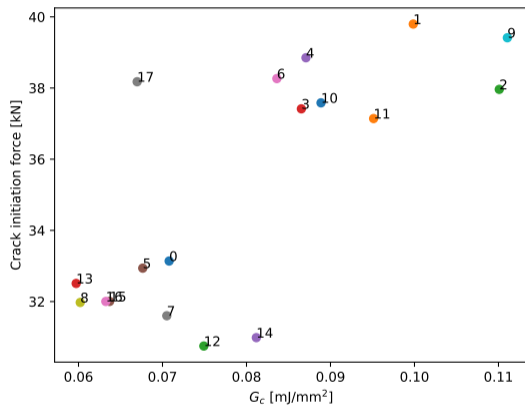
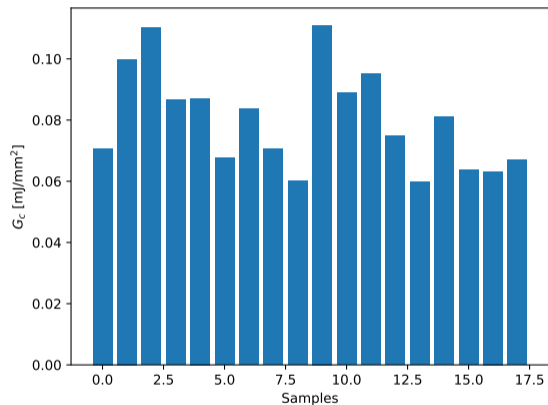
$$\begin{cases} \Delta u_y = -24 \times 10^{-5} \text{ mm} & (\max(\phi) \leq 0.2) \\ \Delta u_y = -6 \times 10^{-5} \text{ mm} & (\max(\phi) > 0.2) \end{cases}$$

[Nguyen et al. 2020] split with
AT1 regularization.

$$\min_{G_c \in \mathbb{R}^+} J(G_c, \ell_0 = L/100)$$



The model does not reproduce the softening behavior observed experimentally, but is capable of reproducing crack initiation!

Identified critical energy release rate G_c 

Without sample #17: $\underline{G_c} = 0.081$ [mJ/mm²] and $\delta_{G_c} = 19.83$ % which is consistent with the classical values available in the scientific literature [Daudeville 1999; Pech et al. 2022b].

Conclusions

Conclusions

- Development of a numerical simulation method to study crack propagation in spruce specimens under compression.
- Identification of the **elastic** and **fracture** material properties from experimental results.
- **The model does not reproduce the softening behavior observed experimentally.**
- **The model reproduces the crack initiation!**

On-going works

- Modeling the observed experimental dispersion using a probabilistic approach for the material properties p and G_c .

Future works

- Reproduce the observed (ductile) softening behavior.
- Simulate the fracture observed on the scale of a furniture assembly.

Thank you for your attention!

You can contact me at matthieu.noel@univ-eiffel.fr.

To obtain further information, access the data or reproduce the identification process, please refer to the following sources:








EasyFEA python software [\[Noel 2024\]](#)









the data and scripts [\[Noel et al. 2024\]](#)

To be submitted in *Engineering Fracture Mechanics* in a couple of weeks.

Références I

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