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# Multi-attribute, Multi-class, Trip-based, Multi-modal Traffic Network Equilibrium Model: Application to Large-scale Network

Mostafa Ameli, Jean-Patrick Lebacque, and Ludovic Leclercq

**Abstract** Transportation systems are dynamically driven not only by non-linear interactions between the different components but also by significant feedbacks between network state and user decision. In this work, we consider a trip-based multi-modal approach to network equilibrium. We assume that mode and path choice is carried out at the same level, therefore travel time depends on the travel path and the mode attributes of travelers. First, we analyze the existing approaches in the literature to model users' heterogeneity. Second, we present a formulation for static traffic network equilibrium and propose a hybrid formulation of the cost function for trip-based traffic assignment. Third, we consider dynamic traffic assignment (DTA) and propose a variational inequality formulation of the trip-based fixed demand function for the multi-class Dynamic traffic equilibrium problem. Finally, we analyze the equilibrium in a large-scale DTA test case (Lyon 6e + Villeurbanne) by a simulation-based approach. Moreover, we propose a novel trip-based algorithm to solve the discrete DTA problem and compare it with the gap function-based method.

## 1 Introduction

Network equilibrium is a situation of the traffic network in which all users' travel costs in all routes actually used are equal and less than those which would be experienced by a single user on any unused route. Generally, travelers consider different criteria in selecting their optimal properties of travel, including travel route and travel mode(s). Most multi-class flow models classify travelers by identifying trav-

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eler attributes. In the literature, this classification process identifies eight categories of information about travelers:

1. Choice model: Users are divided to the different groups based on the behavioral model that they take into account to choose the route and mode(s) [6].
2. Cost function: link cost or path cost can be different for each class of users. e.g. the toll of the links can be different for each class [2].
3. Economic attributes: Users are categorized into different classes based on their income [8].
4. General cost function: The goal of the generalized cost function is to present function which aggregates time and monetary cost. Typically, the general cost function ( $GC$ ) integrates Travel Time ( $TT$ ) and Travel Cost ( $TC$ ).  $GC$  can take into account the class-dependent Value of time (VOT) [10]:

$$GC = TC + \alpha_i.TT \quad (1)$$

where  $\alpha_i$  is VOT for class  $i$ .

5. Knowledge level of the network: users can be classified by the ability to access the real-time traffic information (ATIS) [2].
6. Risk taking attitude: Risk is defined as the expected value of the unfavorable outcome. Travelers treat travel time variability as a risk in their travel choices because it introduces uncertainty for an on-time arrival at the destination [9].
7. Social class: To classify users into social classes cannot make difference in equilibrium calculation. The advantage is to estimate the demand in traffic assignment problems with variable demand [1].
8. Vehicle class: In multi-modal traffic network, vehicle classes are defined in order to calculate the travel time and cost based on mode attributes.

Traffic assignment mathematical models are divided into two main categories: flow-based and trip-based [6]. Here, we consider the trip-based approach to be more realistic for a large-scale traffic network. Therefore the path flow model will be discrete. Concerning the mathematical model, the problem is known as Static Traffic Assignment (STA) when the Origin-Destination (OD) matrix and the link flows are assumed to be time-independent. On the other hand, if time dependence is considered, the problem becomes a Dynamic traffic assignment (DTA). In this study, we apply the model to a real test case (Lyon 6e + Villeurbanne) by a simulation-based approach. In order to solve DTA model, we present a new heuristic approach to find equilibrium and compare it to one of the efficient approach based on Method of successive average (MSA) by Lu et al. [4].

In next section, the multi-class model is presented. The Wardrop condition for the network equilibrium model is discussed for STA and DTA in Sect. 2. Moreover, the uniqueness of the solution is also discussed in this section. The new algorithm to find the network equilibrium and the numerical experiment are explained in Sect. 3. Finally, we conclude this study by presenting the numerical results and discuss them in Sect. 4.

## 2 The Multi-attribute, Multi-class, Trip-based, Multi-modal Model

In order to describe the traveler behavior, a general cost function that takes into account four types of information (categories 2, 3, 4 and 7) defined in Sect. 1, are used. Typically the general cost function ( $GC$ ) will take into account the class-dependent Value of time (VOT) [10] denoted  $\alpha_i$  for class  $i$ , and a vector  $\beta_i$  of economic and social class parameters. Hence the general cost of path  $p$  ( $p \in P$ : set of paths) is:

$$GC_p(\alpha_i, \beta_i) = TC_p(\beta_i) + \alpha_i \cdot TT_p \quad (2)$$

The vector of parameters  $\beta_i$  includes the mode(s) and social attributes of a traveler. For instance, one class in the network can be students who pay less to use the transit network and who have the specific value of time. The value of parameters and classes identification will be presented in Sect. 3. Each user is able to perceive the travel cost and travel time on a link. The demand of OD pair  $w$  ( $w \in W$ : set of OD pairs) is:

$$d_w^i = \sum_{p \in P_w} \pi_p^i \quad , \quad d_w^i \geq 0 \quad (3)$$

where  $i$  is the user class index ( $i \in 1, 2, \dots, k$ ),  $P_w$  is a set of paths for  $w$  and  $\pi_p^i$  is the number of users of class  $i$  on path  $p$ . After assigning the user to the path between OD pair  $w$ ,  $\pi_a$  (the number of users on link  $a$ ) is:

$$\pi_a^i = \sum_{p \in P} \pi_p^i \delta_{ap} \quad , \quad \pi_a = \sum_{i=1}^k \pi_a^i \quad (4)$$

where  $a$  is the link index ( $a \in A$ : set of links in the network),  $\pi_a^i$  is the number of users of class  $i$  on link  $a$  and  $\delta_{ap} = 1$  if path  $p$  contains link  $a$  and 0, otherwise. The travel time/cost function depends on the congestion of the link. Therefore, the travel time and cost are calculated by the following formulas:

$$TT_a = TT_a(\pi_a) \quad , \quad TC_a^i(\beta_i) = TC_a^i(\beta_i, \pi_a) \quad (5)$$

Consequently, the link generalized cost function is calculated as follows:

$$GC_a(\alpha_i, \beta_i) = TC_a(\beta_i, \pi) + \alpha_i \cdot TT_a(\pi) \quad (6)$$

### Static Traffic Network Equilibrium Conditions:

In the STA, the elastic OD demands are assumed to depend on averaged generalized OD travel costs via the OD demand functions. The OD demand functions are general and can depend not only on travel disutilities of each class on pair  $w$ , but also on other classes of users on other OD pairs. Therefore, the demand of class  $i$  for OD pair  $w$  is

$$d_w^i = \Delta_w^i(\overline{GC}_w^i) \quad (7)$$

where  $\overline{GC}_w^i$  is averaged general cost of OD pair  $w$ .  $\Delta_w^i$  denotes the demand function. The static traffic network equilibrium condition is defined as: For each class  $i$ , for all OD pair  $w \in W$  and for all paths  $p \in P_w$ , the equilibrium conditions are

$$\begin{cases} GC_p(\alpha_i, \beta_i) = GC_w^{i*} & \text{if } \pi_p^{i*} > 0 \\ GC_p(\alpha_i, \beta_i) \geq GC_w^{i*} & \text{if } \pi_p^{i*} = 0 \end{cases} \quad (8)$$

$$\begin{cases} \Delta_w^i(GC_w^{i*}) = \sum_{p \in P_w} \pi_p^{i*} & \text{if } GC_w^i > 0 \\ \Delta_w^i(GC_w^{i*}) \leq \sum_{p \in P_w} \pi_p^{i*} & \text{if } GC_w^i = 0 \end{cases} \quad (9)$$

Where  $GC_w^{i*}$  is the minimum general cost of user in class  $i$  for travel between OD pair  $w$  and  $\pi_p^{i*}$  is the optimal number of user class  $i$  on path  $p$ .

As presented in [5] the flow pattern  $\pi^*$  being the solution of this network equilibrium model is equivalent to solving the following finite-dimensional variational inequality formulation:

$$\begin{aligned} & \sum_{i=1}^k \sum_{a \in A} GC_a^i(\pi_a) (\pi_a^i - \pi_a^{i*}) - \sum_{i=1}^k \sum_{w \in W} GC_w^{i*} (d_w^i - d_w^{i*}) \\ & + \sum_{i=1}^k \sum_{w \in W} (d_w^{i*} - \Delta_w^i(GC_w^{i*})) (GC_w^i - GC_w^{i*}) \geq 0 \quad \forall \pi \in \mathcal{H} \end{aligned} \quad (10)$$

Where  $(\pi_a^{i*}, d_w^{i*}) \in \mathcal{H}$  satisfy the equilibrium.  $\mathcal{H}$  denotes the flow constraints (Eq. (3));  $\pi^*$  must also be in  $\mathcal{H}$ . Eq. (10) equivalently, in standard form:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0 \quad \forall X \in \mathcal{H} \quad (11)$$

where  $F \equiv (GC(), -GC())^T, d - \Delta(GC)$  and  $X \equiv (\pi, d, GC)$ .

### Dynamic Traffic Network Equilibrium Conditions:

Consider a network  $G(N, A)$  with a finite set of nodes  $N$  and a finite set of directed links  $A$ . The demand is time-dependent. The period of interest (planning horizon) of duration  $H$  is discretized into a set of small time intervals indexed by  $\tau$  ( $\tau \in T = \{\tau_0, \tau_0 + \sigma, \tau_0 + 2\sigma, \dots, \tau_0 + M\sigma\}$  and  $\tau_0 + M\sigma = H$ ).  $\sigma$  is the duration of the time intervals. In an interval  $\tau$ , travel time and traffic conditions do not change. The important notations to introduce the dynamic equilibrium model are as follows:

$P_{w, \tau}$ : set of paths for  $w$  in departure time  $\tau$ .

$P_{w, \tau}^*$ : set of shortest paths for  $w$  in departure time interval  $\tau$ .

$p$ : index of path,  $p \in P_{w, \tau}$ .

$p^*$ : index of shortest path,  $p^* \in P_{w, \tau}^*$ .

$GC_{p, \tau}(\alpha_i, \beta_i)$ : general cost of path  $p$  for user class  $i$  in departure time  $\tau$ .

$GC_{w, \tau}^{i*}$ : minimum general cost of OD pair  $w$  for user class  $i$  in departure time  $\tau$ .

For each class  $i$ , for all OD pair  $w \in W$  and for all paths  $p \in P_w$ , the dynamic traffic

network equilibrium condition with fixed travel demands for the aforementioned traffic network equilibrium problem is

$$\begin{cases} GC_{p,\tau}(\alpha_i, \beta_i) - GC_{w,\tau}^{i*} \geq 0 \\ \pi_{p,\tau}^i (GC_{p,\tau}(\alpha_i, \beta_i) - GC_{w,\tau}^{i*}) = 0 \\ \pi_{p,\tau}^i \geq 0 \end{cases} \quad (12)$$

The solution of this problem is equivalent to finding the solution to the following variational inequality:

$$\sum_{w \in W} \sum_{\tau=1}^T \sum_{i=1}^k \sum_{p \in P(w,\tau)} GC_{w,p,\tau}^{i*} (\pi_{w,p,\tau}^i - \pi_{w,p,\tau}^{i*}) \geq 0 \quad (13)$$

Existence of solution(s) to Eq. (13) needs the continuity of the generalized cost functions of paths with respect to the numbers of users on paths. In this case, the uniqueness of the solution requires the strict monotonicity of the OD paths generalized cost functions with respect to OD path flows (i.e. number of cars). In general urban transportation networks, these conditions are not guaranteed to hold. In next section, we present a new heuristic algorithm to find a solution for the dynamic traffic network equilibrium model.

### 3 Trip-based Solution Algorithm and Numerical Experiments

This simulation-based study works on trip-based DTA which transforms the solution space from continuous space to discrete space. The optimization process is executed for each departure time interval contains two loops. The outer loop finds a new shortest path from origin to destination  $w$  and the inner loop finds a best assignment path for the path set of the current outer loop [7]. At the end of each outer loop, the best solution is kept and used to enter to the next outer loop. The optimization method updates the assignment plan in each iteration of inner loop. There is a convergence test for each loop. There are two differences between Outer loop Convergence Test (OCT) and Inner loop Convergence Test (ICT). First, in the outer loop we calculate the quality indicator (Eq. (14)), based on the new shortest path(s) but in ICT the path set is fixed during the inner loop and just the shortest path is updated in each iteration. Second, in ICT we calculate the difference of total gap between two iterations of the inner loop, but in OCT, we compare the total gap with a small reference value to check the convergence. The total gap as a quality indicator in time interval  $\tau$  is:

$$Gap(\pi, GC^*) = \sum_{w \in W} \sum_{i=1}^k \sum_{p \in P(w,\tau)} \pi_{w,p}^i (GC_{w,p}^i - GC_{w,p}^{i*}) \quad (14)$$

The heuristic methods are path-swapping descent direction methods that decompose the problem into many (origin-destination, departure time interval) subproblems and solve each of them by adjusting time-varying OD flows between all non-shortest paths and the shortest path(s). Moreover, in this study, the optimization process is applied to each class of users.

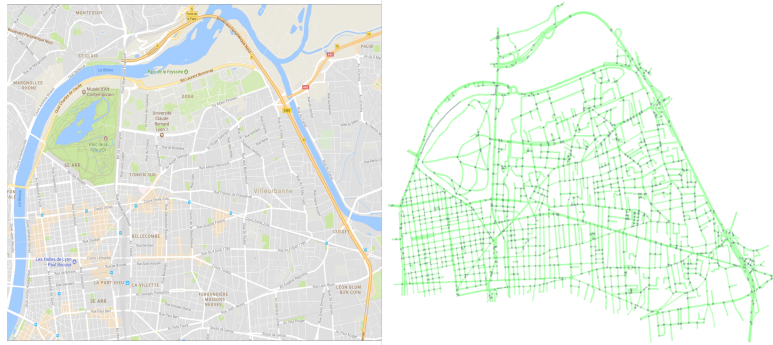
To enable the deployment of large-scale (simulation-based) DTA models, [4] proposes a heuristic approach which we called "Gap-based" method in this study. In iteration  $\lambda$  of this method, a fraction  $\frac{1}{\lambda+1}$  of users on each non-shortest path swap to the one of the shortest path in current iteration. The new method which is called "Probabilistic method" is a trip-based approach which is without step size for optimization. For each user the probability of swap to the shortest path(s) is calculated by following formula:

$$\text{Swap indicator} = \frac{GC_p - GC_p^*}{GC_p} \quad (15)$$

Then, we use a random number for taking a decision about swapping the user or not. If the random number is lower than indicator the user swaps to the shortest path and otherwise, the user keeps his/her path for the current iteration.

The model for multi-class DTA is applied to a large-scale network of Lyon 6e + Villeurbanne with 1,883 Nodes, 3,383 Links, 94 Origins, 227 Destinations and 54,190 trips. This network located in Lyon is described in Fig. 1. The Trips are divided into 9 classes that are presented in Tab. 1.

In this work, we use Symuvia<sup>1</sup> as a trip-based simulator for calculating the needed variables in the network. Symuvia has been developed by the LICIT laboratory in IFSTTAR. It is a microscopic simulator based on a Lagrangian discretization of the LWR model [3]. It has a simulation time-step equal to 1 second and collects the information at the link level every 1 minute. Note that the dynamic simulator uses Eq. (6) to calculate the generalized travel cost on a link at each time interval.



**Fig. 1** Lyon 6e + Villeurbanne: Mapping data ©Google 2017 and the traffic network using by Symuvia

<sup>1</sup> Symuvia is an open source simulator that will be available starting winter 2018.

**Table 1** User classes and their value of time for the Lyon 6e + Villeurbanne network

Percentage of trips	Class name (job)	Value of time[euro/sec]
44.7%	worker	0.01
8.2%	student	0.005
0.7%	worker-student	0.008
18.8%	retired	0.006
3.2%	at home	0.004
8.2%	elementary school	0.004
10.3%	secondary school	0.005
5%	unemployed	0.004
0.9%	other	0.006

The optimization process is executed for 9 outer loops that means we have 10 paths per OD. In each outer loop, 20 inner loops have been executed to find the best assignment pattern. Note that we have one more simulation per outer loop for keeping the best assignment pattern.

## 4 Numerical Results and Conclusion

The full optimization process is done for both methods and the quality of the solution at the end of each outer loop is evaluated by the following indicator that calculates the travel time gap per user in minutes:

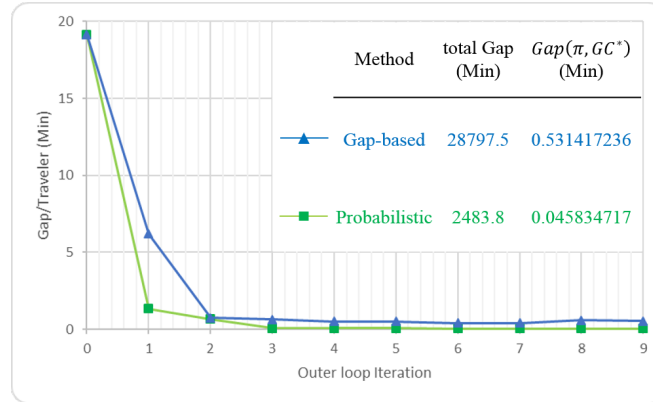
$$Gap(\pi, GC^*) = \frac{\sum_{w \in W} \sum_{\tau=1}^T \sum_{i=1}^k \sum_{p \in P(w, \tau)} \pi_{w, p, \tau}^i (GC_{w, p, \tau}^i - GC_{w, p, \tau}^{i*})}{\sum_{w \in W} \sum_{\tau=1}^T \sum_{i=1}^k \sum_{p \in P(w, \tau)} \pi_{w, p, \tau}^i} \quad (16)$$

The test results of two optimization methods on the multi-modal large-scale network are presented in Fig. 2. The total gap (Eq. (14)) and the quality indicator (Eq. (16)) are calculated for each outer loop with the updated shortest path. The average delay for each user by the Gap-based method is around 30 sec but this value for the Probabilistic method is around 3 sec. It shows that the new method obtains 10 times better quality than the Gap-based method on this large-scale network.

The multi-attribute multi-class model and optimization process has a successful application on a multi-modal large scale network. The new method (Probabilistic) as a trip-based approach works significantly better than the Gap-based method. It is free from step size and it converges faster in this study. The authors propose the following topics for the future works:

- Extend the multi-attribute, multi-class trip-based model to activity-based models in order to consider travel with activity chains;
- Compare and analyze the performance of the method in other large-scale networks with different topologies;
- Compare the other types of heuristic methods with the Probabilistic method.





**Fig. 2** Comparison of Gap-based method and Probabilistic method which is proposed by this study

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