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► **To cite this version:**

Mostafa Ameli, Sérgio Batista, Monica Menendez. Simulation-based Dynamic System Optimum on regional networks considering aggregated traffic models. 12th International Scientific Conference on Mobility and Transport: Mobility Innovations for Growing Megacities, TUM, Apr 2022, Singapore, Singapore. hal-04493034

HAL Id: hal-04493034

<https://univ-eiffel.hal.science/hal-04493034>

Submitted on 6 Mar 2024

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Simulation-based Dynamic System Optimum on regional networks considering aggregated traffic models

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March 6, 2024

1 Introduction

The aggregated traffic models based on the Macroscopic Fundamental Diagram (MFD) (Daganzo, 2007, Geroliminis & Daganzo, 2008, Jin, 2020) present promising prospects to mimic the network-wide dynamics with a low computational cost. These models require the partitioning of urban networks into regions (Batista et al., 2021b), which permits to define an aggregated or regional network. In each region, vehicles travel at the same average speed and the dynamics are regulated by an MFD. The MFD reflects the relationship between the region's mean speed and the accumulation of vehicles. Exchange flows between the different regions represent the traffic dynamics in the network. The application of these aggregated traffic models range from control strategies (Yang et al., 2018, Sirmatel & Geroliminis, 2019) to perimeter control with route guidance (Yildirimoglu et al., 2018), multi-modal systems control (Haitao et al., 2019, Paipuri & Leclercq, 2020, Ameli et al., 2019) and traffic management (Batista & Leclercq, 2019, Ambühl et al., 2021, Chen et al., 2021), as some examples. Johari et al. (2021) presents a good review of the MFD applications and future challenges.

One crucial component of any traffic model is the assignment of travelers in the network. In urban networks, vehicles are assigned to routes that are represented by a sequence of links with a fixed physical length. The case of regional networks is slightly different, as it requires the identification of paths on the regional network (Batista et al., 2021a), and the characterization of their explicit distribution of travel distances (Batista et al., 2019, Ameli et al., 2022). A path in the regional network is represented by the ordered sequence of traveled regions between the Origin (O) and Destination (D) regions. These are two main differences compared to conventional assignment modules in urban networks.

Batista & Leclercq (2019) proposed a framework to target the Dynamic User Equilibrium on regional networks, where the travel time of a path p , TT_p^{OD} , is defined by the ratio between the explicit distribution of travel distances (L_{rp}) and the evolution of the mean speed in the regions ($v_r(n_r)$):

$$TT_p^{OD} = \sum_{r \in X} \frac{L_{rp}}{v_r(n_r)} \cdot \delta_{rp}, \forall r \in X \quad (1)$$

where r is a generic region; X represents the set of regions defining the regional network, $r \in X$; n_r is the region's accumulation; and δ_{rp} is a binary variable that equals 1 if path p travels on region r , or 0 otherwise.

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1 Generally, traffic assignment models aim to calculate System Optimum (SO) based on systems ob-
 2 jectives and/or users goals. Mathematically speaking, at the network level, the equilibrium problem is
 3 equivalent to minimizing the travel time for each path, while the SO model minimizes the marginal travel
 4 time of each path (Ameli, 2019). Both problems are well-defined and studied in urban networks (Wang et al.,
 5 2018). However, in the regional networks, even though some studies have focused on the network equilibrium
 6 conditions (Yildirimoglu & Geroliminis, 2014, Laval et al., 2018), little attention has been paid to the System
 7 Optimum. The reason is the complexity behind the calculation, Only Yildirimoglu et al. (2015) discussed
 8 a perimeter control application with route guidance that targets the Dynamic System Optimum. However,
 9 the work did not consider the concept of paths as introduced by (Batista et al., 2021a), nor explicit travel
 10 distances (Batista et al., 2019) or the explicit calculation of the path marginals. In this paper, we propose
 11 a methodological framework to calculate the Dynamic System Optimum on regional networks, considering
 12 MFD-based traffic models. We propose an explicit formulation to calculate the path marginals based on the
 13 network dynamics. We showcase the application of this methodology on a test network, and briefly analyze
 14 the network dynamics compared to the User Equilibrium conditions.

15 Simulation-based Dynamic System Optimum on regional networks

16 Under the System Optimum (SO) conditions of the network, drivers' routes are assigned such that the total
 17 travel time (TTT) of the whole system is minimized. Mathematically, we can describe the SO in terms of
 18 travel time minimization as the following nonlinear constrained optimization problem:

$$\begin{aligned} \min_{\vec{Q}} TTT &= \sum_{r \in X} TT_r \\ &= \sum_O \sum_D \sum_{r \in X} \sum_p \frac{L_{rp}}{v_r(n_r)} \delta_{rp} \end{aligned} \quad (2a)$$

19
 20 s.t.:
 21

$$\sum_p Q_p^{OD} = Q^{OD}, \forall (O, D) \in W \quad (2b)$$

$$Q_p^{OD} \geq 0, \forall (O, D) \in W \quad (2c)$$

22 where \vec{Q} represents the vector that contains the flows of all paths of the regional network; TT_r is the total
 23 travel time of a region r ; Q_p^{OD} represents the path flow traveling on path p that connects the OD pair;
 24 Q^{OD} represents the demand of the OD pair; and W is the set of all regional OD pairs of the network. We
 25 note that in the calculation of the travel times in Equation 2a, one can consider the distribution of travel
 26 distances L_{rp} or an average travel distance \bar{L}_{rp} . Equation 2b represents the demand conservation equation
 27 of each regional OD pair. Equation 2c represents the non-negativity of the flows traveling on path p that
 28 connects the OD pair.

29 The accumulation n_r of a generic region r is related to the accumulation of all paths p (n_{rp}) that
 30 travel on this region:

$$\begin{aligned} n_r &= \sum_p n_{rp} \cdot \delta_{rp} \\ &= \sum_O \sum_D \sum_p Q_p^{OD} \cdot Q^{OD} \cdot \delta_{rp} \end{aligned} \quad (3)$$

31 Due to paths correlations, the calculation of the path marginals is not trivial. Even in urban networks,
 32 usually, the path marginals are computed by surrogate functions that aggregate the approximation of link
 33 marginals (Peeta & Mahmassani, 1995). The only study of link marginals at the macroscopic level has

1 been done by [Yildirimoglu & Kahraman \(2018\)](#). They used GPS records with map-matching to estimate
 2 link marginals on OD graph without calculation of SO. Here, we formulate the path marginals on regional
 3 networks. The marginal travel time, $T_p^{marginal}$, of path p is:

$$T_p^{marginal} = TT_p + \frac{\partial TT_p}{\partial n_{rp}} \cdot n_{rp}, \forall p \in \Psi \quad (4)$$

4 where TT_p represents the travel time of path p (see Eq. 1); and Ψ is the set of all paths in the regional network.
 5 In this paper, we consider only average travel distances \bar{L}_{rp} .

6 Using the chain rule, we can express the derivative in Eq. 4 as:

$$\begin{aligned} \frac{\partial TT_p}{\partial n_{rp}} &= \frac{\partial TT_p}{\partial v_r} \cdot \frac{\partial v_r}{\partial n_r} \cdot \frac{\partial n_r}{\partial n_{rp}} \\ &= - \sum_{r \in X} \frac{\bar{L}_{rp}}{v_r^2(n_r)} \cdot \frac{\partial v_r}{\partial n_r} \cdot \delta_{rp} \end{aligned} \quad (5)$$

7 We determine the Dynamic System Optimum equilibrium using a quasi-dynamic approximation,
 8 where we split the total simulation period T into smaller intervals δt . We determine the network equilibrium
 9 for each of these intervals during which the path flows are maintained constant. This quasi-static approx-
 10 imation permits to capture the evolution of the traffic dynamics and changes in demand over time [Ameli
 11 et al. \(2018\)](#). Therefore, we calculate the derivative term $\frac{\partial n_r}{\partial n_{rp}}$ in Eq. 5 as:

$$\frac{\partial v_r}{\partial n_r} = \frac{v_r(n_{rp}(t + \Delta t)) - v_r(n_r(t))}{n_r(t + \Delta t) - n_r(t)} \quad (6)$$

12 Note that, Eq. 6 is determined based on the MFD dynamics on the regional network.
 13 Substituting Eq. 6 and Eq. 5 in Eq. 4, the path marginals become:

$$T_p^{marginal} = \sum_{r \in X} \frac{\bar{L}_{rp}}{\bar{v}_r(n_r)} \delta_{rp} - \frac{\bar{L}_{rp}}{\bar{v}_r^2(n_r)} \cdot \frac{v_r(n_r(t + \Delta t)) - v_r(n_r(t))}{n_r(t + \Delta t) - n_r(t)} \cdot n_{rp} \cdot \delta_{rp}, \forall p \in \Psi \quad (7)$$

14 As one can observe, the path marginal became a negative term in Eq. 4. We now analyze the
 15 consistency of this equation. First, recall that the MFD assumes that all vehicles travel at the same average
 16 speed inside a given region r . This average speed decreases as the accumulation of vehicles in the region
 17 increase and vice-versa. We consider three scenarios in this analysis:

- 18 1. The demand on path p is constant. This implies that $\frac{\partial v_r}{\partial n_r} = 0$, and the path marginal travel time
 19 becomes $T_p^{marginal} = \sum_{r \in X} \frac{\bar{L}_{rp}}{\bar{v}_r(n_r)} \delta_{rp}$. Then the SO and UE conditions of the network are equivalent
 20 under constant demand.
- 21 2. The increase of the demand on path p (n_{rp}), leads to a reduction on the region's mean speed $v_r(n_r)$.
 22 This implies that $\frac{\partial v_r}{\partial n_r} < 0$ and the path marginals become an additive term in Eq. 4.
- 23 3. The decrease of the demand on path p (n_{rp}), leads to an increase on the region's mean speed $v_r(n_r)$
 24 which in turn also leads to $\frac{\partial v_r}{\partial n_r} < 0$. Therefore, the path marginal is also an additive term in this case
 25 in Eq. 4.

26 We determine the network equilibrium using the classical Method of Successive Averages, with a
 27 descent step of $1/j$, where j is the descent iteration. The network equilibrium is reached when the *Gap*
 28 ([Ameli et al., 2020b](#)) is inferior to 0.1 or when the number of violations $N(\phi)$ ([Sbayti et al., 2007](#)) is inferior
 29 than 0.01. The *Gap* acts as an indicator of how far the network is from the equilibrium conditions ([Ameli
 30 et al., 2020a](#)). The number of violations indicate how many paths have a difference of path flows between
 31 two consecutive descent iterations superior to a pre-defined threshold. We also set as a stopping criterion a
 32 maximum of 100 descent iterations.

1 Preliminary results

2 Figure 1 (a) depicts the test network that consists of the 3rd and 6th districts of Lyon and the city of
3 Villeurbanne (L63V) (Ameli et al., 2020c, Batista & Leclercq, 2020). This network has 1883 nodes and 3383
4 links, and is partitioned into seven regions. Figure 1 (b) shows the calibrated MFD functions for each region.
5 Figure 1 (c) shows the demand scenarios taken into account, which consists of a total of 7 OD pairs and 21
6 possible paths. We have calibrated the travel distances following the procedure described in Batista et al.
7 (2019). We consider a total simulation period of $T = 3$ [h], and a time interval $\delta t = 300$ [s]. We utilize an
8 accumulation-based MFD model (Mariotte et al., 2017, Mariotte & Leclercq, 2019) to model the network
9 dynamics (Amelia et al., 2021).

10 We determine the Dynamic System Optimum and User Equilibrium (following the methodology de-
11 scribed in Batista & Leclercq (2019), Ameli et al. (2021)) on this test network and demand scenarios. Figure 2
12 shows the evolution of the accumulations in the regions as a function of the simulation time t [hr]. The blue
13 lines represent the results for the User Equilibrium (UE), while the green lines show the ones of the System
14 Optimum (SO). As one can observe the SO assignment permits to reduce the congestion in the regions
15 compared to the UE case, as expected. The selfish behavior of drivers for their path selection, increases the
16 total travel time of the system, degrading its performance. Also, the price of anarchy (PoA) for the scenario
17 depicted in Figure 2 is 1.0362, which confirms that the performance of the system is improved under System
18 Optimum conditions, as previously discussed.

19 This paper formulates the SO conditions for aggregated networks and MFD-based models. As a next
20 step, we will incorporate this framework on a control strategy that mitigates the network-wide emissions,
21 and investigate how the travelers' rationality influences the controller's performance.

22 Acknowledgments

23 The authors acknowledge support by the NYUAD Center for Interacting Urban Networks (CITIES), funded
24 by Tamkeen under the NYUAD Research Institute Award CG001 and by the Swiss Re Institute under the
25 Quantum CitiesTM initiative. The opinions expressed in this article are those of the authors alone and do
26 not reflect the opinions of CITIES or its funders.

27 The authors also thank the laboratory LICIT (Univ Eiffel/ENTPE) for sharing the test network.

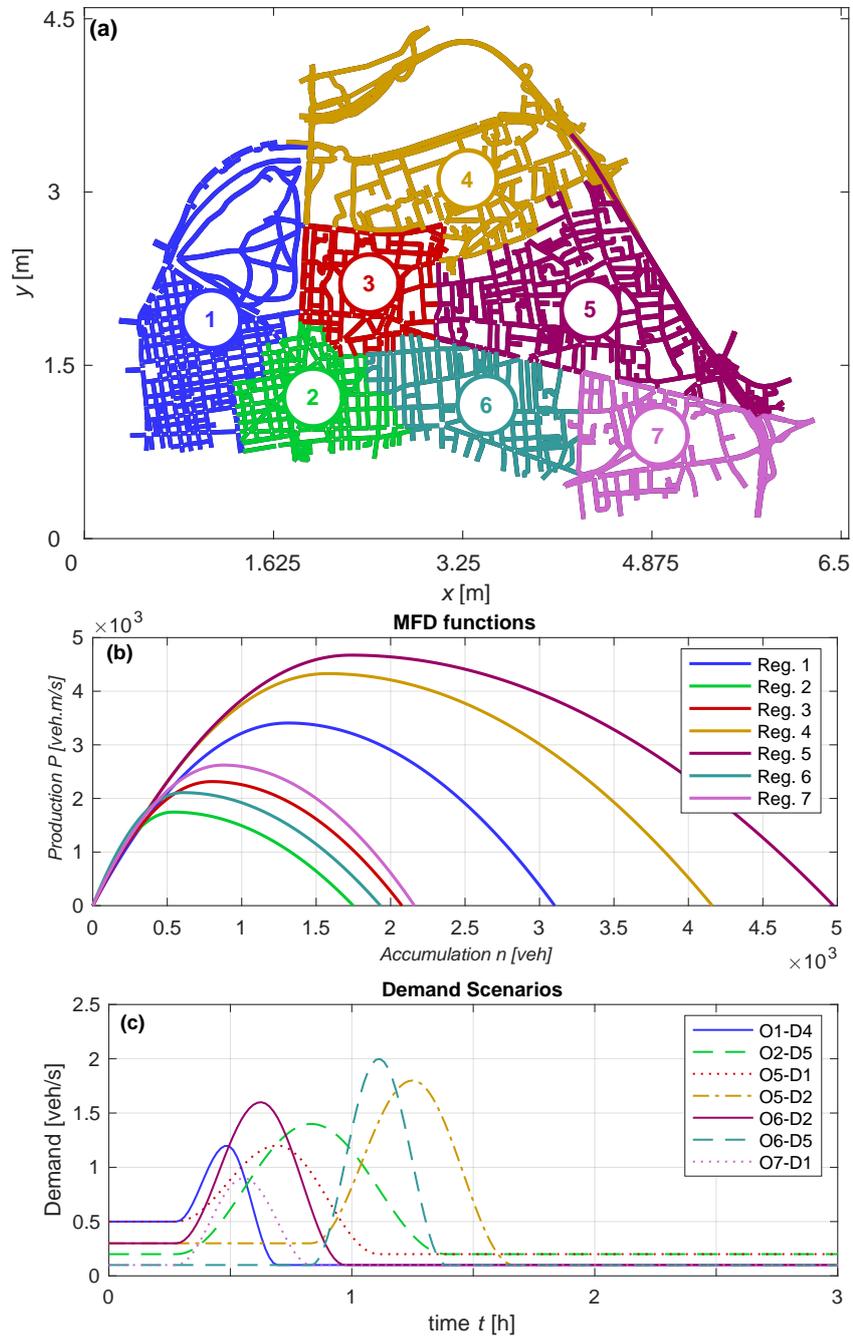


Fig. 1 – (a) L63V network. (b) Production MFD functions. (c) Demand scenarios as a function of the simulation time t [hr].

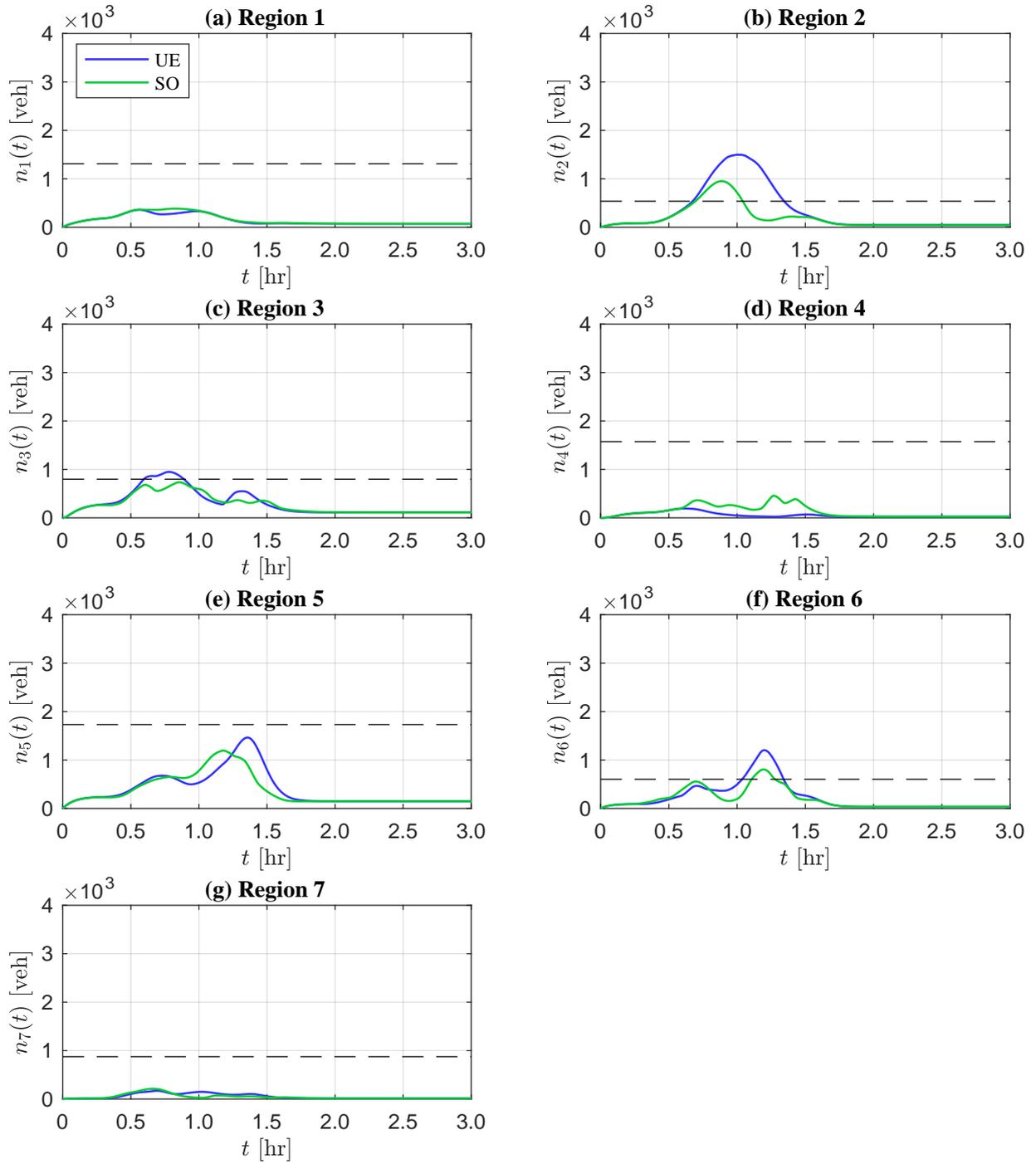


Fig. 2 – Evolution of the accumulation n_r [veh] as a function of the simulation time t [hr] for all seven regions. The results are depicted for the User Equilibrium (UE) and System Optimum (SO) conditions of the network. The horizontal black dashed line represents the critical accumulation of each region.

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