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# Position error modelling for sensitivity analysis of GNSS road applications

Based on Ifsttar-GEOLOC and Q-Free cooperation

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Abstract— Road transport, after Location-Based Services, is the second application domain of GNSS. Measuring the end-to-end performance of intelligent transportation systems (ITS) – and how this depends on the embedded GNSS chipset – makes the core of the on-going European standardization initiative at CEN-CENELEC TC5/WG1. This article first presents the methodology, called "Sensitivity analysis", onto which the future norm will rely. Then the problem of modelling the GNSS positioning error is addressed, with some practical solutions, and the choice of one in particular, which is developed in an experimental context for validation purpose. The signal processing made from the full scale experiment in order to figure out the parameters of the model is presented. The use of this model in a simulation process is shown and analysed, so that first conclusions are drawn and research perspectives designed.

Keywords—GNSS positioning; intelligent transportation systems; error modelling

#### I. INTRODUCTION

Given the principle of GNSS positioning – whose performances are highly influenced by the conditions of the operational environment – and the need to ensure appropriate performances throughout the lifetime of the equipment, the development of standards and certification on positioning performance is fundamental for device vendors and service providers, especially when it concerns safety- and liability-critical applications.

In this context, European standardization organisations aim to produce standards for the use of geo-positioning services for navigation and localisation systems, especially in the road transport domain, which is the second application domain of GNSS, after Location-Based Services (LBS).

The development of such standards will certainly have a positive impact on future road ITS industry developments, especially in regard to the need to provide industry with the highest practicable degree of uniformity in the provision and operation of GNSS services [1].

One of these standardization organisations, the CEN-CENELEC TC5/WG1, proposed in its 1<sup>st</sup> European standard [2] a general methodology for assessing the conformity of a road transport application or a complete road ITS system, using a given GNSS-based positioning terminal.

This methodology, called "Sensitivity analysis", relies on the straightforward assumption that a road ITS system is composed of 2 main sub-systems, which are: the positioning terminal (using GNSS, but not always the only technology) and the application module that transforms the raw Position, Velocity and Time (PVT) data from the terminal into "Application quantities" necessary to provide the final ITS services to the users. For instance, the system "Personal Navigation Device" comprises a positioning terminal (the GNSS chipset) and an application that performs mapmatching, attributes retrieval and driver information through the man-machine interface. The final service is the guidance of the driver.



Figure 1: Simplified architecture of a positioning-based road ITS system

The Sensitivity analysis is based upon field tests in real conditions of the GNSS-based positioning terminal (GBPT) to identify a PVT error model that will be used to generate automatically a high number of synthetic degraded trajectories that will be processed by the application module. From the outputs of the application module, using the end-to-end performance metrics (also called KPIs for Key Performance Indicators), will be assessed the end-to-end performance of the system, depending on the performance of both the GBPT and the application itself.

This method offers the advantage to duplicate field tests executed in real operational conditions, the only ones capable of capturing the reality of the physical phenomena, in order to run a high number of trials, necessary to assess performances that are generally expressed by low probabilities.

The sensitivity of the system to the performance of the GBPT can be analysed by increasing artificially step by step the amplitude of the errors of the PVT error model until the moment when the target E2E performance is no longer fulfilled. Figure 2 represents the general principle of the Sensitivity analysis.

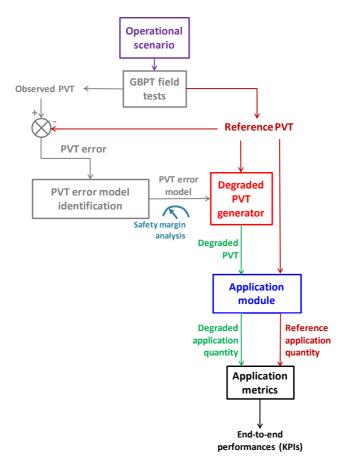


Figure 2: Sensitivity analysis general principle

The present paper is about the position error modelling and more precisely about its application to a real set of experimental data that has not been used for the establishment of the model but for figuring out its specific parameters.

#### II. STATE OF THE ART

In the GNSS domain, the community has produced several degradation (or error) models applying to:

- Radio signal-in-space, before signal tracking,
- Pseudorange, phase and Doppler errors, also called raw data, where most literature references concentrate,
- Position, Velocity and Time, after raw data have been processed for navigation purpose.

In this article, and within SaPPART action, road transport applications are targeted. Therefore, degradation models that refer to GNSS receiver Position output (and Velocity and Time to a lesser extent), are of key interest. Very few literature references address the question of directly modelling the error in the position domain, contrary to the pseudorange domain in particular.

Classically, the GNSS position errors are considered as white noise (e.g. in Extended Kalman Filters that perform loose coupling with odometry and/or inertial measurement unit integration).

More advanced models consider that these errors are coloured: their autocorrelation has not a Dirac shape, but rather a triangular shape, indicating significant correlation times. These correlation times are due to correlation in the pseudorange errors from which the positions are computed, and from the computation itself (least-squares, but more often Kalman filtered processes are implemented in black-box-like receivers, for which no implementation and filter tuning details are given).

For instance, in [3] the authors fuse odometry, GPS positions and camera vision of lane markings in a Kalman filter using such position error model.

#### III. SAPPART MODELS

#### A. First modelling proposed within SaPPART

Important research efforts in GPS PVT error models have been carried out recently within the COST action SaPPART by Ifsttar and GMV researchers [4].

Several models have been proposed and assessed based on an experimental dynamic data set (uBlox LEA6-T receiver). The models use the local Frenet frame (see Figure 3), tangent to the trajectory of the vehicle. The error is decomposed either into its along-track and cross-track components or into its local polar components, radius and angle. The local frame, whose orientation is that of the trajectory, has interesting properties concerning the error. Its orientation is actually also that of the travelled street, where buildings create multipath, and across which the positioning error is known to be larger than along [5].

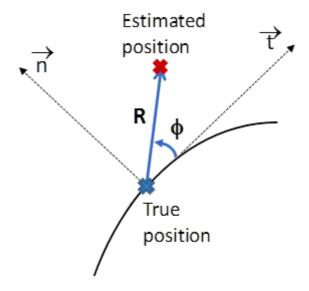


Figure 3: The local Frenet frame

Among these models, let us focus on the two followings:

- CDF-based: along and cross track errors are randomly drawn (and weighted the one with respect to the other) from part of the data set used for learning, and correlated with time using a filter parameterized with the speed of the vehicle.

- Laplace-Cauchy-based: errors are considered in a local polar frame, the radius being modelled by piecewise constant values Laplace distributed, changing with a probability p, on top of which is added a random walk from a Cauchy distributed noise. The angle is a random walk from a Cauchy distributed noise too. Truncatures are also applied on the distributions.

The probability p and the parameters of the statistical distributions have to be learnt from experimental data.

Last, low-pass filters and normalization constants have to be designed.

Details can be found in [6].

The CDF-based method needs only two parameters to be adjusted: the correlation parameter between along-track and cross-track, and the parameter for the time filtering. For the Laplace-Cauchy based method, the set of parameters to adjust is larger: parameters for the probability density functions of the radius and the angle, for the filters and for the normalization constants.

From the results presented in [6], the autocorrelation of the error signal cannot be reproduced using the CDF-based method. The Laplace-Cauchy-based method seems to have better results: both the autocorrelations and CDFs are close to the true ones and the signals look similar to the actual signals from the uBlox LEA6-T receiver.

#### IV. FIRST EXPERIMENTAL VALIDATION OF THE LAPLACE-CAUCHY-BASED METHOD

A first experimental validation on a data set in Frankfurt, Germany, with uBlox LEA5-T receiver, was conducted during summer 2015 by Q-Free and Ifsttar [8]. It included the identification of the parameters, the simulation of error series, and their comparison to the true error. This data set was provided by Q-Free.

Generating the error signal in the radius consists of several steps. First a piecewise constant basis made of steps that are generated using a truncated Laplace distribution:

$$A'[n+1] = \begin{cases} A[n] \text{ with probability } (1-p) \\ L_r[n] \text{ with probability } p \end{cases}$$
 (1)

where A'[n+1] is the radius signal with no noise,  $L_r[n]$  is a realization of a truncated Laplace distribution giving the height of the new step, and p is the probability of a new step. Then, an additive noise, which is accumulated over each step, is added:

$$A'[n+1] = A[n+1] + C_r[n]$$
 (2)

where  $C_r[n]$  is a realization of a truncated Cauchy distribution. To clarify, the accumulation of the Cauchy noise does not operate over the whole signal, but it is accumulated only during one step. It is reset when there is a new draw from the Laplace distribution. The values of the parameters to generate  $L_r[n]$  and  $C_r[n]$  samples are estimated by means of Bayesian identification methods [7] assuming Gaussian

distributions, with refinements made by trials and errors. Finally, the signal is filtered using an auto-regressive filter that relates the expected error signal with the simulated signal.

The angle is generated using a random walk given by:

$$\phi[n+1] = \phi[n] + S_{\phi}[n] \tag{3}$$

where  $S_{\phi}[n]$  is a realization of a Cauchy distribution, before the signal is low-pass filtered. The values of the parameters to generate  $S_{\phi}[n]$  samples properly are determined experimentally.

The data set is composed of 28 trajectories from Frankfurt, Germany, see Figure 4. The drive route goes through deep urban canyons with dense traffic, comprising 4.9 km, with average driving speed of 15 km/h. It is without tunnels and covered sections.



Figure 4: Drive route in Frankfurt recorder by the refernce receiver

The reference track is provided by a Novatel CPOS combined with an IMU by applying kinematic Precise Point Positioning (PPP) post processing using Terrapos software. The vehicle is equipped with 8 low-cost GNSS receivers of various models from uBlox, ST, and SiRF in addition to a camera providing photographic evidence of the driving environment. The equipment is shown on Figure 5. On the roof one can see the antenna for the reference NovAtel receiver. In the front window one can see the test devices. In this article, only the low cost receiver uBlox LEA5-T is considered, but a similar study could be made on the other GNSS receivers.



Figure 5: test vehicle from Q-Free

Manual inspection of the recorded trajectories reveals that 22 trajectories show good GNSS performance from the uBlox receiver, 4 trajectories show significantly larger errors, while 2 trajectories contain large error peaks of 400 meters for a duration of up to 30 seconds (see Figure 6).

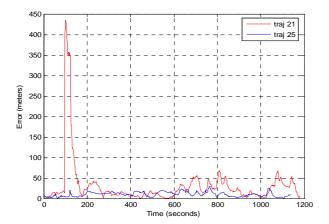


Figure 6: Position error of best (25) and worst (21) trajectories

The parameters were estimated for all trajectories using the Hutter method [7] implemented by Monsifrot.

The Hutter method is a Bayesian exact regression of piecewise constant functions. It operates for Gaussian distributions of both the identified steps (their mean  $\mu$  and standard deviation  $\sigma$  are identified) and the noise applying above them (this noise is centred and its standard deviation  $\sigma_{\text{noise}}$  is identified as well). A similar optimization algorithm does not exist for Laplace distributed steps and, a fortiori, with the accumulated Cauchy noise. This is the reason why, after running the Hutter algorithm and getting initial values for simulating a Laplace-Cauchy-based model, there is an adjustment of the set of parameters to achieve manually, by trials and errors.

The Gauss, Laplace and Cauchy distributions are defined by their respective probability density function:

$$f_{Gauss}(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$
 (4)

$$f_{Laplace}(x) = \frac{1}{2b} e^{\frac{|x-\mu|}{b}}$$
 (5)

$$f_{Cauchy}(x) = \frac{1}{\pi \gamma} \frac{1}{\left(1 + \frac{(x - x_0)^2}{\gamma^2}\right)}$$
 (6)

By analogy, the mean value  $\mu$  from Hutter is kept the same as the initial mean value for the Laplace simulated signal. Similarly, the parameter b is fixed initially at  $\sigma$ . Since the Cauchy noise is accumulated in the simulation, its parameter  $\gamma$  must be much lower than the standard deviation  $\sigma_{\text{noise}}$  of the additive noise of the Hutter identification (otherwise the error would grow much too much within the duration of a step), and this is typically to be fixed in simulation by trials and errors.

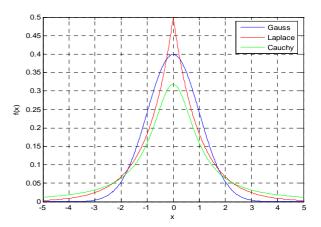


Figure 7: The standard Gauss, Laplace and Cauchy distributions  $(\mu=x_0=0$  and  $\sigma=b=\gamma=1)$ 

When simulating, the analysis of the signal produced is visually inspected with regard to the original signal time series. In addition, the CDFs are compared, as well as the autocorrelations.

Despite the Hutter method is exact, the complete process leading to the final parameters is an approximation, with empirical results. Trajectory 25 was, after thorough analysis, chosen as the best data set to estimate an outline of the complete set of parameters. Figure 8 shows the result of a Hutter derived estimation of step probability where the stepwise red line is the approximation.

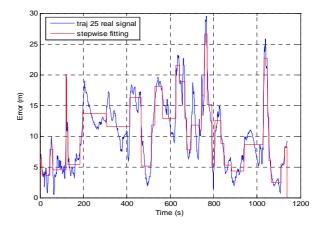


Figure 8: Hutter estimation of stepwise approximation and step probability

The filter applied to the radius has been modelled by an autoregressive moving average with exogenous inputs (ARMAX), and identified with the stepwise approximation as signal input, and the original error as signal output. The number of poles (here: 2) and zeroes (and: 0) must be determined by trial and error. Moreover, a dead time of 1 sample (1 second) was applied and the noise disturbance was pink (it used 1 previous value). To summarize, the Matlab armax function was called with [na=2, nb=1, nc=1, nk=1] parameters. The trial and error approach is judged successful when the residuals of the ARMAX identification are uncorrelated. This can be checked with the resid Matlab function.

The filter applied to the angle is here empirically a moving average of the 10 previous input values.

Table I shows the obtained parameters.

TABLE I. SIMULATION PARAMETERS

Parameter	Selected parameter
Laplace	•
(radius)	
μ	11 m
b	36 m
min	0.1 m
max	22 m
Cauchy	
(radius)	
X <sub>0</sub>	0
γ	0.5 m
min	-1.0 m
max	1.0 m
Filter coef	{ 1, -1.728, 0.7544 }
(radius) a	
Filter coef	0.02518
(radius) b	
Step	0.032
probability p	
(radius)	
Cauchy	
(angle)	
$\mathbf{x}_0$	0
γ	0.0161 rad
min	-pi
max	pi
Filter coef	1
(angle) a	
Filter coef	{ 0.1, 0.1, 0.1, 0.1,
(angle) b	0.1, 0.1, 0.1, 0.1, 0.1,
	0.1 }

Radius errors were simulated using the selected parameters within the model. One simulation result is shown in Figure 9 along with trajectory 25. The autocorrelation can be seen in Figure 10. It is somewhat lacking at the second lobes, but the simulation is quite good as can be seen in Figure 11 where the CDFs of the simulated signal and the targeted signal are close.

Figure 12 shows a tentative to reproduce trajectory 21. For this purpose, the upper bound of the truncated Laplace distribution has been moved from 22 m to 400 m, all other parameters being unchanged.

A peak is a sudden very big error that has a relative short duration. The big error around  $t=100\,\mathrm{s}$  is very sudden, but it lasts for some epochs, this lasting effect of the error is probably due to filtering in the GNSS receiver. In the simulated error signal, this lasting effect is not completely satisfying. The correlation is not matching well and neither is the CDF (not shown). There is obviously a model mismatch: large transient peaks are still not properly reproduced.

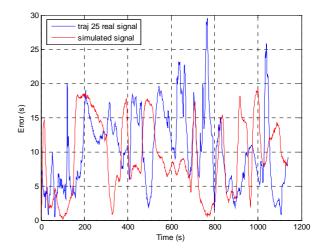


Figure 9: Time evolution of simulated and target signals (trajectory 25)

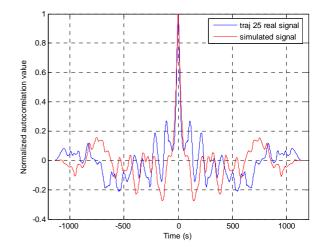


Figure 10: Autocorrelation function of signals (trajectory 25)

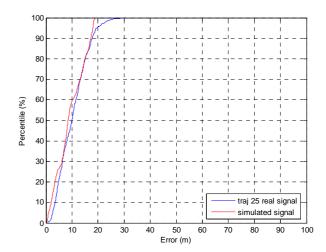


Figure 11: Cumulative distributions of signals (trajectory 25)

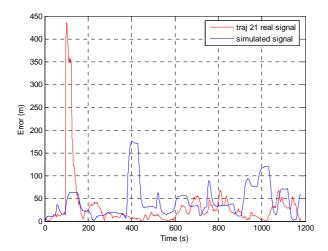


Figure 12: Time evolution of simulated and target signals (trajectory 21)

#### V. ANALYSIS OF THE RESULTS OBTAINED

#### A. Parameter estimation

The model has a number of parameters for the error radius simulation. These relate to the error magnitude step probability (p), to the Laplace distribution parameters  $(\mu, b, m)$  min and max), to Cauchy distribution parameters  $(x_0, \gamma, m)$  min and max), and to the filter coefficients (a and b). The Hutter derived method, currently implemented in a Matlab script, estimates the parameters for the Laplace distribution and step probability quite well. However, because of a model mismatch between the Cauchy noise-based simulation and the Hutter implementation, the parameter  $\gamma$  is currently not properly estimated. The simulation accumulates the Cauchy random samples during one step, whereas the parameter estimation model assumes that the error is not accumulated. To the best of the authors' knowledge in signal processing and identification,  $\gamma$  must be fitted manually.

#### B. Effect of stops and slow traffic flow

As the trajectories are from downtown Frankfurt, stops are frequent, caused by red lights and traffic jams. This requires further attention, in particular when reconstructing the alongand cross-track errors. Actually, in case of a large radius error, this error should not rotate if stopped. The angle of the error vector must be limited taking into account the vehicle speed.

#### C. Exceptional large position errors

A challenging issue is with the peaks, i.e. gross outlying errors. The poor fit of the simulation results is due to the fact that the error model does not have, at this moment, the ability to model peaks. The simplest modification to the model would be to add a small chance of having a very big error at every point. The occurrence of such peaks and their magnitude, frequency and duration should be tuned. In the Frankfurt data set, the peaks occurred twice (at the same geographical location), and have a duration of 20-25 seconds, this accounts for approximately 0.4% of the total driving time (total over all 28 trajectories).

#### D. Conclusions

Suggesting for GNSS position error modelling – and applying to real data – a Laplace-Cauchy-based model has improved the understanding of the simulation method, its performances and limitations. Several parameters cannot be estimated by the existing identification tools, and need to be manually adjusted. The algorithm should be refined to the extent that it becomes more automatic, but the distributions handled being non-Gaussian, it seems that identification tools well suited still do not exist in the signal processing community.

Although the method needs more development, it has been found that the model, with parameters estimated from one trajectory, is representative for most trajectories, tuning only the upper truncature bound of the Laplace distribution, except for those with peaks. The modelling was easier due to the shift from cross- and along-track errors to radius and angle. The study has shown that the model is able to work well in challenging urban GNSS environment with typically urban driving patterns with frequent start/stop. Future work is needed to refine the error model for peaks which occur on start-up and after exiting tunnels for example.

#### **Acknowledgment**

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