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► **To cite this version:**

Reza Shahin, Pierre Hosteins, Paola Pellegrini, Pierre-Olivier Vandanjon. A full factorial sensitivity analysis for a capacitated Flex-Route Transit system. 2023 8th International Conference on Models and Technologies for Intelligent Transportation Systems (MT-ITS), Jun 2023, Nice, France. pp.1-6, 10.1109/mt-its56129.2023.10241378 . hal-04416930

**HAL Id: hal-04416930**

**<https://univ-eiffel.hal.science/hal-04416930v1>**

Submitted on 31 Jan 2024

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# A full factorial sensitivity analysis for a capacitated Flex-Route Transit system

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**Abstract**—Popularity has increased for semi-flexible transit systems because they combine the flexibility of a Demand Responsive Transit with the affordable fare of Conventional Public Transport. In this study, we investigate a Flex-Route Transit: a shuttle performs several trips in a pre-defined service area; it follows a main route between origin and destination terminals and must pass through some mandatory checkpoints along this route. Between any consecutive checkpoints, an available slack time is available for shuttles to deviate from the main route to pick-up and drop-off customers. We implement a Mixed-Integer Linear Programming formulation from the literature with an added shuttle capacity constraint. We use it to perform a sensitivity analysis of the system using a full factorial experimental design over three parameters, namely: shuttle capacity, level of demand and available slack time. The combined influence of the parameters is analyzed using the method called Analysis of Variance.

**Index Terms**—Flex-Route Transit, On-demand transportation, sensitivity analysis.

## I. INTRODUCTION

Demand Responsive Transit (DRT) is intended for door-to-door transport of individuals. DRT primary objective was to provide a non-profit pick-up and drop-off service for elderly and handicapped people [10]. DRT provides a high level of service to its customers that includes: 1. door-to-door pick-up and drop-off; 2. flexible service schedule; 3. short travel time. Nevertheless, this service is often costly.

In contrast, Conventional Public Transportation (CPT) works according to a predetermined timetables and picks-

up and drops-off customers at predetermined stations. Such system is far less expensive than DRT, but it does not offer the same quality of service in terms of flexibility. Due to the rising awareness for global warming, demand and therefore congestion, transport authorities are attempting to minimize the use of private cars [11]. To do this, they aim to increase the quality of service of shared alternatives, by offering cost-effective, flexible public transportation. Flex-Route Transit (FRT) has evolved in order to strike a compromise between fare policy and perceived customer convenience. The FRT system merges the flexibility of DRT with the affordability of CPT. As in CPT, in FRT a shuttle performs several trips between terminals, visiting intermediate stops *checkpoints* in each trip (See Fig. 1).

Customers may use the FRT system as a regular CPT one, by boarding and disembarking at checkpoints. However, if there is sufficient slack time, the shuttle may deviate from the base route to perform door-to-door pick-up and drop-off services for clients within the predefined service area. The shuttle is expected to always proceed ahead towards the destination depot, unless approved to go back by a maximum threshold distance known as *backtracking*. The FRT system can accommodate four different client types depending on their pick-up or drop-off location [4]: 1. PD (regular): pick-up and drop-off are at checkpoints; 2. PND (hybrid): pick-up is at checkpoint, drop-off is not at checkpoint; NPD (hybrid): pick-up is not at checkpoint, drop-off is at checkpoint; NPND (random): pick-up and drop-off are not at checkpoints. We formally define a *request* as the following elements, pertaining to a specific customer: a pick-up and a drop-off time, as well

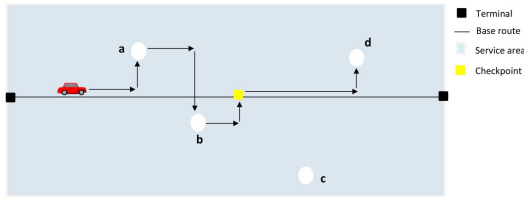


Fig. 1. Explanation of how FRT works

as a pick-up and a drop-off point. For a better understanding of the FRT system, an example is provided in Fig. 1, where all the requests are known in advance (static environment). There are two factors that impact whether a customer is picked-up or not in the first trip after the request is issued: availability of shuttle capacity and slack time. A lack of either of these resources means that one or more customers will be assigned to a later trip. Fig. 1 shows an example in which the shuttle deviates from the base route to pick-up and drop off customers at points ‘a’, ‘b’ and ‘d’. Nevertheless, due to the lack of available slack time or capacity, customer ‘c’ will have to be assigned to a later trip.

In the early 2000s, the Metropolitan Transit Authority (MTA) in Los Angeles County adopts such a system for the first time on line 646 [1]. This system operates during the day shift as classic CPT, and during the night shift as FRT. This shift from CPT to FRT happens during the night shift for two reasons. On the one hand, nighttime demand is generally low, and enabling shuttles to detour from the main route does not increase the riding time of on-board customers significantly. On the other hand, at night walking is often unsafe and few transfer possibilities are available. The fact that inclement weather makes walking to the closest checkpoint difficult is a further factor that makes FRT appealing as testified by its subsequent introduction in Canada and China [2], [3].

Several Mixed Integer Linear Programming (MILP) formulations have been developed to tackle the vehicle scheduling in FRT systems [1], [4], [5], [7], [8]. However, only limited investigations on the sensitivity of the system to its parameters have been carried out. Specifically, “One-Factor-at-a-Time” (OFAT) is the sole approach used to perform sensitivity analysis. In this approach, one parameter is allowed to vary within a certain range while the others are held constant. The parameter is then reset to its default value and another parameter is varied.

In this study, we apply a more advanced statistical method to achieve a clearer understanding of the sensitivity of the FRT objective function when many parameters are simultaneously changed. Specifically, we apply Analysis of Variance (ANOVA) to investigate the links and impacts shuttle capacity, level of demand and available slack time, exploiting a full factorial experimental design.

The remainder of the paper is organized as follows. We review the literature in Section (II) and provide the MILP formulation in Section (III). The methodology and the results

of our sensitivity analysis are presented in Sections IV and (V), respectively. We conclude in Section (VI).

## II. LITERATURE REVIEW

The FRT was first presented in the literature as Mobility Allowance Shuttle Transit (MAST) [1] and afterwards renamed to FRT [3]. To maintain consistency, FRT is used throughout the article. The literature addresses the scheduling of vehicles in FRT systems through exact, heuristic, and meta-heuristic algorithms.

The first work on FRT in the literature was introduced by [1] in 2004. The authors proposed the first MILP formulation to address the problem while minimizing: 1. distance traveled by a shuttle; 2. riding time of clients; 3. waiting time of clients. A comparison is made between the FRT system and CPT based on the following four criteria: 1. riding time; 2. walking time; 3. waiting time; 4. shuttle distance traveled. The measured performance indicates that FRT performs better than CPT. In [4], the model of [1] is further developed by adding three sets of valid inequalities to improve the linear relaxation of the model. It is found that the valid inequalities reduce the computation time by up to 90%. However, some of the largest instances still cannot be solved in less than 10 hours. In [5], the authors seek to understand the critical number of customers to switch from one to two shuttles. For this purpose, two methodologies are employed to address the problem: a MILP formulation extending the one of [4] to handle a fleet of shuttles; and an approximate analytical model to validate the results. The results of the MILP show that with more than 14 customers/hour, it is beneficial to switch to two shuttles. A sensitivity analysis is also performed over the weight of shuttle travel time in the objective function. It shows that if this weight increases, the threshold on the number customers also increases. In [6], the authors consider a maximum time (upper bound) at which a customer can be picked-up: they add time windows constraints and shuttle capacity constraints to the model of [1]. The provided system is then compared with a CPT system in which customers use a shared biking system to get to the closest checkpoint stop to be picked-up. It is found that the two systems perform similarly. Nevertheless, based on customers’ perspective, the FRT system is considered to be a more convenient service. A sensitivity analysis is performed over the level of demand and number of checkpoints. The results show that for low demand neighborhoods with a low number of checkpoints, an FRT system performs better. In [7], the authors introduce some Meeting Points (MPs) within the service area, so if the pick-up or drop-off location of customers cannot be inserted in the schedule, they can walk to or from the closest MP as long as MPs are within an acceptable walking distance. A MILP formulation extending the one of [1] is provided, and the waiting time in the objective function is replaced by the walking time. A Memetic Algorithm is developed to solve the large scale instances. The results show that for more than 12 customers/hour the rejection rate is reduced up to 20% by employing a MP strategy. A sensitivity analysis is also

References	Year	# of shuttles	Capacity	# of Customers
[1]	2004	1	×	25/hour
[4]	2008	1	×	17/hour
[5]	2011	1-2	×	14/hour
[7]	2019	1	×	25/trip
[6]	2021	1	50	50/trip
[8]	2021	10-30	10	45/trip
<b>Current study</b>	2023	1	15-20-25	40-60-80/hour

TABLE I  
SUMMARY OF THE LITERATURE.

References	RTW	DL	NC	MP	AWD	ST	SC
[5]	✓						
[6]		✓	✓				
[7]				✓	✓		
<b>Current study</b>		✓				✓	✓

TABLE II  
SUMMARY OF SENSITIVITY ANALYSIS IN THE LITERATURE.  
RTW (RIDING TIME WEIGHT); DL (DEMAND LEVEL); NC (NUMBER OF CHECKPOINTS); MEETING POINTS (MP); AWD (ACCEPTABLE WALKING DISTANCE); ST (SLACK TIME); SC (SHUTTLE CAPACITY).

performed over the number of MPs and acceptable walking distance for customers. It is found that by increasing them, more customers can be served. In [8], the authors seek to reduce shuttle operation costs by employing Modular Autonomous Vehicles (MAVs), which are small vehicles that can be assembled into larger ones. In FRT, MAVs can split between any consecutive checkpoints to handle non-PD customers. A MILP formulation is developed to address the problem, which is intractable in practice. A customized dynamic programming algorithm is developed to handle the route scheduling, and a fast heuristic solves the customers' assignment to each MAV. Different scenarios including 10 to 30 vehicles and 15 to 45 customers are tested. It is found that using MAVs can reduce the operational costs significantly.

We provide a summary of the literature in Table I. The FRT system is designed to provide service in low demand neighborhood, justifying the fact that few papers in the literature consider shuttle capacity constraints. Recently, two papers [6], [8] take into account shuttle capacity of 50 and 10, respectively, when modeling the FRT. As shown in Table II, the level of demand is only assessed in [6] when performing a sensitivity analysis.

### III. MATHEMATICAL MODELING

To study the impact of parameter variations on the FRT performance, we implement the MILP formulation of [1] with an added shuttle capacity constraint. The problem is modeled on a graph  $G = (N, A)$ , with  $N = \{0, 1, 2, \dots, n\}$  the set of nodes and  $A$  the set of arcs between pairs of nodes. We have the following assumptions: 1. the shuttle starts and ends each trip at depot; 2. the shuttle performs several trips; 3. customers are not rejected due to the lack of slack time or shuttle capacity, but are assigned to a later trip(s). In the MILP, we use the following notation:

- $K_{PD}/K_{PND}/K_{NPD}/K_{NPND}$  = set of PD/ PND/ NPD/ NPND requests, with  $K = K_{PD} \cup K_{PND} \cup K_{NPD} \cup K_{NPND}$ .
- $K_{HYB}$  = set of hybrid requests (PND and NPD types).
- $ps(k) \in N$  = pick-up nodes of  $k$ ,  $\forall k \in K \setminus K_{PND}$ .
- $ds(k) \in N$  = drop-off nodes of  $k$ ,  $\forall k \in K \setminus K_{NPD}$ .
- $N_0$  = set of checkpoints stops.
- $N_n$  = set of non-checkpoint stops.
- $N = N_0 \cup N_n$ .
- $o$  and  $d$ : origin and destination depots.
- $A$  = arcs.
- $R$  = number of trips.
- $C$  = number of checkpoints.
- $TC = (C - 1) \times R + 1$  = total number of stops at checkpoints in the schedule.
- $TS = TC + |K_{PND}| + |K_{NPD}| + 2|K_{NPND}|$  = total number of stops.
- $\delta_{i,j}$  = rectilinear travel time between  $i$  and  $j$ ,  $\forall i, j \in N$ .
- $q_i$  = the load picked-up ( $> 0$ ) or dropped-off ( $< 0$ ) at each node  $\forall i \in N$ .
- $Q$  = shuttle capacity.
- $\theta_i$  = scheduled departure time of checkpoint stop  $i$ ,  $\forall i \in N_0$ , ( $\theta_1 = 0$ ).
- $\tau_k$  = ready time of request  $k$ ,  $\forall k \in K$ .
- $b_i$  = service time for stop  $i$ .
- $w_1/w_2/w_3$  = objective function weights.

#### Variables

- $x_{i,j} = 1$  if an arc  $(i, j) \in A$  is used by the shuttle.
- $t_i$  = departure time from stop  $i$ ,  $\forall i \in N$ .
- $\bar{t}_i$  = arrival time at stop  $i$ ,  $\forall i \in N \setminus \{1\}$ .
- $p_k$  = pick-up time of request  $k$ ,  $\forall k \in K$ .
- $d_k$  = drop-off time of request  $k$ ,  $\forall k \in K$ .
- $Q_i$  = shuttle load when leaving node  $i \in N$ .

The objective function to minimize is modeled as:

$$w_1 \sum_{(i,j) \in A} \delta_{i,j} x_{i,j} + w_2 \sum_{k \in K} (\bar{t}_{ds(k)} - t_{ps(k)}) + w_3 \sum_{k \in K} (t_{ps(k)} - \tau_k) \quad (1)$$

subject to the following constraints:

$$\sum_{i \in N} x_{i,j} = 1, \quad \forall j \in N \setminus \{o\} \quad (2)$$

$$\sum_{j \in N} x_{i,j} = 1, \quad \forall i \in N \setminus \{d\} \quad (3)$$

$$t_i = \theta_i, \quad \forall i \in N_0 \quad (4)$$

$$t_{ps(k)} \geq \tau_k, \quad \forall k \in K \quad (5)$$

$$\bar{t}_{ds(k)} \geq t_{ps(k)}, \quad \forall k \in K \quad (6)$$

$$\bar{t}_j \geq t_i + x_{i,j} \delta_{i,j} - (1 - x_{i,j})M, \quad \forall (i, j) \in A \quad (7)$$

$$t_i \geq \bar{t}_i + b_i, \quad \forall i \in N \setminus \{o, d\} \quad (8)$$

$$Q_j \geq Q_i + q_j - (1 - x_{i,j})M, \quad \forall (i, j) \in A \quad (9)$$

$$\max\{0, q_i\} \leq Q_i \leq \min\{Q, Q + q_i\}, \quad \forall i \in N \quad (10)$$

$$x_{i,j} \in \{0, 1\}, \quad \forall (i, j) \in A \quad (11)$$

$$t_i, \bar{t}_i \geq 0, \quad \forall i \in N \quad (12)$$

The objective function (1) is the weighted sum of three different parts, namely the total shuttle time traveled, the total travel time of all passengers and the total waiting time of all passengers. Here waiting time is the time gap between the passengers ready time and the actual pick-up time. Network Constraints (2) and (3) allow each stop (except at node  $o$  and  $d$ ) to have exactly one incoming and one outgoing arc, so that all stops will be performed. Constraints (4) force the departure times from the checkpoints to be fixed, while Constraints (5) prevent each pick-up stop from having its departure time earlier than its ready time. Constraints (6) prevent the drop-off stop to be scheduled earlier than the pick-up stop for each request. Constraints (7) enforce that for each traversed arc  $(i, j)$ , the arrival time at node  $j$  should be no less than the departure time from node  $i$  plus the time needed to travel between  $i$  and  $j$ . The last term ensures that for any  $x_{ij} = 0$  the constraint becomes trivially satisfied. These constraints also guarantee that every feasible solution does not contain inner loops, but a single route from node  $o$  to node  $d$ , visiting all nodes in  $N$  only once. Constraints (8) link arrival and departure times for each stop  $i$  in the graph. Constraints (9) and (10) compute the load variables according to the arcs used in the solution and ensure that the shuttle capacity is respected.

#### IV. METHODOLOGY

In the literature, a few studies have investigated the effect of changing the value of a single parameter on the objective function, by employing the OFAT method explained in the Introduction ([5], [6], [7]). In this paper, we aim at understanding the impact of the values of multiple parameters, both in isolation and jointly. To do so, we exploit a full factorial experimental design. Here, we consider various values for each parameter (respectively called *levels* (values) and *factor* in the ANOVA terminology), and we run the MILP with all their combinations (which are called *effects*). For instance, with three parameters and two values for each, this leads to perform  $2^3 = 8$  runs.

We assess the results of the full factorial experiments design through ANOVA (Analysis of Variance). ANOVA is a frequently used statistical approach for evaluating hypotheses [9] that can accommodate a great number of experimental designs. It considers the effects of qualitative variables (in our case, the problem parameters), on one output. The main reason why ANOVA is useful is that it can provide a comprehensive understanding of the interaction impact between parameters on the output. This helps us to understand if the impact of one parameter changes depending on the value of the other parameters.

ANOVA relies on a statistical model which compares differences in means. In order to determine if the means between different groups of data are significantly different (which points to a significant effect on the output), it uses the variance inside the different groups. A group corresponds to a number of independent measures of the output of the system under study, which corresponds for us to the solutions of the MILP presented above for different FRT instances. To each

Parameters	Values	Units
Number of checkpoints	5	-
Service time	18	seconds
Service area length	10	km
Service area width	2	km
Vehicle speed	30	km/h
PD	10	%
PND	40	%
NPD	40	%
NPND	10	%
$w_1, w_2, w_3$	1	-

TABLE III  
DEFAULT PARAMETER SETTING

combination of parameters therefore corresponds a number of solutions called *repetitions*.

The usual way to display the output of an ANOVA analysis is through what we can call the ANOVA table. This table lists a certain number of properties of the variation of the system output (the objective function for our optimization problem), depending on the combination of input parameters considered. It has one row for each such combination, listed in the column labeled *Effect*. Then, we have the Degree of Freedom in the Numerator (DFn), which is the number of independent factors (variables) in the ANOVA table minus one. The Degree of Freedom in the Denominator (DFd) represents the total number of observations minus the number of groups being compared. The ' $F$ ' stands for the  $F$ -Statistic, which is the ratio of two variances: between-group variance and the within-group variance. The  $F$ -Statistic is used to test the null hypothesis that the means of all groups are equal. The ' $P$ ' stands for the  $p$ -value and is a measure of the probability of observing a test statistic as extreme as the one computed from the sample data, assuming the null hypothesis is true. If  $p$ -value is less than the significance level (less than 0.05), then we reject the null hypothesis and conclude that there is a significant difference between at least two of the group means. The most important parameters for assessing the impact of the different effects are the  $p$ -value (under the column labeled P) and the *generalized eta square* (under the column ges). The magnitude of the impact of the corresponding 'Effect' is provided by the 'ges' column. If the value of 'ges' is less than 0.02 the impact is small, if it is between 0.02 and 0.26, the impact is moderate, whereas a value above 0.26 implies a large impact [12]. The other columns of the table are provided for re-production purposes and we refer the interested reader to [12] for further details.

To perform our sensitivity analysis, we first choose a default parameter setting (See Table III). The percentage of the customers of each type are preserved identical to the one of [1], which is based on the Los Angeles case study. The following studies on the FRT also use the same values for establishing proportions of customers and for fixing the remaining parameters. However, to expand a bit the system size we increase the number of checkpoints to five. We conduct a sensitivity analysis on three parameters: demand level (uniformly distributed in time and space over a 5-

No.	Slack time	Demand	Shuttle capacity
Scenario 1	5	200	15-20-25
Scenario 2	10	200	15-20-25
Scenario 3	15	200	15-20-25
Scenario 4	15	200-300-400	15
Scenario 5	15	200-300-400	20
Scenario 6	15	200-300-400	25

TABLE IV

SCENARIOS FOR ASSESSING OBJECTIVE FUNCTION VALUE AND SHUTTLE CAPACITY THRESHOLD

Effect	DFn	DFd	F	P	P < 0.05	ges
S	2	2673	30254.160	0.00e+00	*	0.958
D	2	2673	165552.544	0.00e+00	*	0.992
C	2	2673	3815.375	0.00e+00	*	0.741
SD	4	2673	4270.673	0.00e+00	*	0.865
SC	4	2673	1344.892	0.00e+00	*	0.668
DC	4	2673	422.110	3.42e-282	*	0.387
SDC	8	2673	180.318	5.50e-244	*	0.351

TABLE V

S (SLACK); D (DEMAND); C (CAPACITY); SD (SLACK:DEMAND); SC (SLACK:CAPACITY); DC (DEMAND:CAPACITY); SDC (SLACK:DEMAND:CAPACITY).

hour time horizon), slack time and shuttle capacity. Each of these parameters has three possible values. Demand levels are 200, 300, and 400; slack times are: 5, 10, and 15 minutes; shuttle capacities are 15, 20 and 25. A full factorial analysis requires then 27 parameter combinations. To perform a robust sensitivity analysis, we solve 100 instances for each parameter combination in which the random demand realization varies.

Furthermore, the specificity of our method is the interaction of any type of parameter combination, which implies that it is not straightforward to generalize a conclusion drawn from an OFAT analysis. We choose to illustrate this phenomenon on combinations of capacity with the other parameters, since this is the first sensitivity analysis including the study of the effect of capacity. We highlight two sets of scenarios, set (a) and set (b). In set (a), we discuss over the results of nine scenarios, that are the combination of slack time and capacity with a fixed level of demand of 200 customers and define three sets of scenarios as outlined in Table IV (scenarios 1-3). We also investigate the combination of demand and capacity in set (b), along similar lines (scenarios 4-6).

## V. DISCUSSION OF THE RESULTS

The results of the ANOVA are shown in Table V, where the columns correspond to the quantities described in Section IV.

As expected, all three parameters have a pretty strong impact on the FRT system. In particular, level of demand has the most significant impact on the system output as shown in Table V, followed by slack time and shuttle capacity, respectively. As we discussed earlier in the previous section, here we first discuss the results of scenarios in set (a).

Each scenario set considers a different value of the slack time at checkpoints. It gathers the values of the objective function for all three values of the shuttle capacity. The average values of the objective function for the resulting nine different combinations of slack time and capacity are displayed

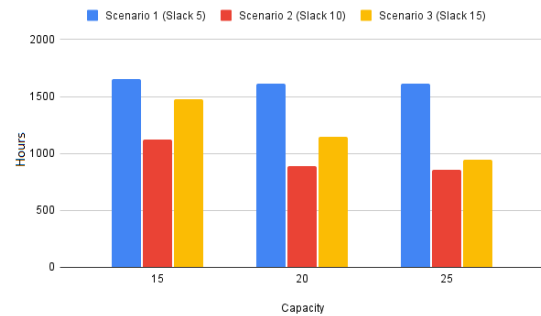


Fig. 2. Total objective function value

in Fig. 2. The general trend is that the objective function decreases as capacity increases. However, the specific behavior depends on the specific slack time scenario considered. If we compare scenarios 1 and 2, we can measure a difference of 528 hours of objective function with capacity 15 and 755 hours with capacity 25, which amounts to a 43% relative increment. This illustrates how differently the system will react to a change in capacity depending on the values of the other system parameters, for example in terms of operating costs.

Moreover, the ratio of objective functions between scenarios 1 and 2 evolves from a value of 1.47 for capacity 15 to 1.88 for capacity 25. As a consequence, it is very difficult to evaluate the system behavior by changing only one parameter at a time, for example starting from a base configuration with a slack time of five minutes and a capacity of 15. In such a case, one can first investigate the effect of a capacity of 25, and then of a slack time of 10 minutes, however, neither the differences nor the ratios are conserved when both parameters evolve. This demonstrates the importance of performing sensitivity analyses considering multiple parameters. The figure also illustrates that the effect of slack time is not monotonic for a given level of demand and that the variation in the system behavior will change by a significantly larger or smaller amount depending on the chosen shuttle capacity.

Next, we investigate the scenarios in set (b), each associated to a different capacity value, as detailed in Table IV (scenarios 4-5-6). What we try to illustrate is a phenomenon already encountered in the FRT literature where the system is said to *saturate* when the demand reaches a critical level. With this level, the shuttle cannot satisfy all the non-PD customer requests as soon as possible. The result is that some requests must be handled at later trips, which increases their waiting time. The fact that non-PD customers tend to accumulate and their waiting time tends to increase signals that the FRT system is not well calibrated with respect to the expected level of demand. We will therefore adopt a customer-oriented view and display the average waiting time per customer, which is only one part of the full objective function of the MILP. The slack time value is fixed at 15 minutes in order to make sure that the saturation phenomenon, if any, comes from a lack of capacity. The results are plotted in Fig. 3 in a similar fashion

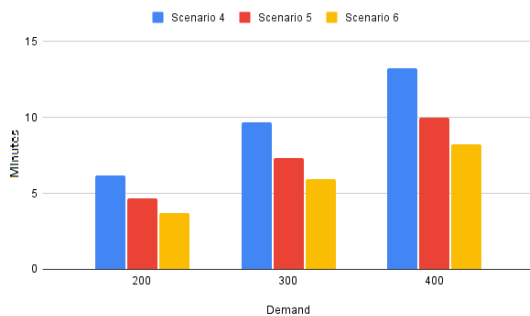


Fig. 3. Average waiting time per customer (minutes)

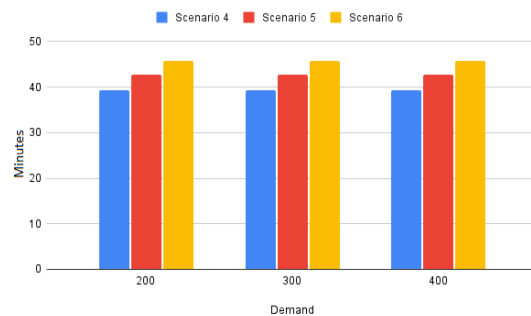


Fig. 4. Average riding time per customer (minutes)

to Fig. 2.

While the waiting time per customer generally increases with the level of demand (which signals that the system is starting to saturate), we can see that the increment value depends on the exact shuttle capacity. While the difference between capacities 15 and 25 is around 2.5 minutes for a demand of 200, it culminates at around 5 minutes for a demand of 400, which represents a 100% relative variation. The fact that we consider an average waiting time *per customer*, as opposed to the *total waiting time*, rules out the natural expectation that it would scale proportionally to the demand level.

The plot can reveal additional information to transport operators on the conditions in which they can provide a guaranteed average level of service to their customers, e.g., an average waiting time of no more than 10 minutes. Interestingly, if we plot instead the riding time per customer for the same configurations of parameters, see Fig. 4, we see a slight increase with respect to the shuttle capacity (as expected since the shuttle can perform more detours to handle more customers within a given trip) but no particular variation when the demand increases. This is a hint that the FRT should also be studied with respect to each of its objectives as an overall variation of the total objective function can hide very different variations of its components.

These results are interesting from a strategic perspective to anticipate the impact of the choice of shuttle depending on the different levels of demand, in order to weigh the investment costs compared to the level of service provided for customers.

## VI. CONCLUSION

In this study, we implemented the MILP formulation of [1] for an FRT system and added a capacity constraint to take into account a finite shuttle capacity. We performed a sensitivity analysis with respect to the shuttle capacity, the level of demand and the slack time at checkpoints, using a full factorial experimental design. We analyzed it using ANOVA in order to assess parameters interdependence on the FRT system behavior.

Our statistical analysis showed that all three parameters have a strong impact, including capacity. We also observed that all parameter combinations have a clear interaction in their

impact, such as the combination of slack time or demand level with the shuttle capacity, and illustrated this effect on the full objective function or some of its components.

In future research, we can refine ANOVA results with more sophisticated sensitivity analysis methods such as Sobol indices [13]. We will also investigate the impact and interactions of a larger number of parameters, such as the frequency departure at the depot when the system can use a fleet of shuttles instead of a single shuttle. Moreover, Design of Experiments (DOE) is a powerful statistical tool used to systematically explore and optimize complex systems. They can offer reduced testing costs, efficient parameter estimation, improved understanding of the system, and robustness analysis.

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