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# Topology optimization to fracture resistance: a review and recent developments

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## Abstract

Topology Optimization (TO) methods for fracture resistance offer new possibilities for designing stronger structures or materials with lower masses than conventional designs. This article presents an overview of topology optimization techniques for fracture resistance, from pioneering works to the most recent developments at the time of writing. We first review stress-based methods, which were the forerunners of crack resistance methods, producing optimal designs that prevent any damage or crack initiation. Other works followed, taking into account the presence of defects or cracks in structures, but using classical approaches aimed at minimizing compliance in an elastic framework. TO methods for fatigue damage are also an important branch of these approaches and are reviewed. We then present more recent methodologies, including non-linear effects in structural design, such as plasticity and damage. Finally, we describe the latest methods of TO design for fracture resistance, including an explicit description of crack propagation during loading, from initiation to failure of structures and materials. In particular, the design of two-phase materials that are more resistant to cracking and that can be manufactured by 3D printing is discussed. The article concludes with some challenges and promising avenues for the coming years in this field.

**Keywords:** Topology optimization; fracture; structures; materials

## 1 Introduction

Topology optimization (TO) is a well-known computational method for designing structures with optimal geometries in order to optimize an objective under possible constraints. Since the pioneering work of Bendsoe and Kikuchi in the late 1980s, the technique has been widely developed and has enjoyed considerable success in the engineering community. Initially applied to mass minimization under compliance constraints or, conversely,

to compliance minimization under volume constraints, TO has since been extended and applied to a wide variety of problems such as mechanical and thermal loads on structures, fluid flow, waste management, structural mechanics and thermics, fluid flow, dynamics, acoustics and biomechanics, among others. (see review articles in [1, 2]).

TO can be categorized into several popular approaches: Solid Isotropic Material with Penalization (SIMP) method [3–5], Bidirectional Evolutionary Structural Optimization (BESO) method [6, 7], Level-Set Method (LSM) [8, 9], Phase Field method [10], Moving Morphable Components (MMC) [11, 12], and their combinations [13]. Review of these methods can be found in state-of-the-art papers [2, 14–18]. An interesting comparison review on these techniques, with advantages and drawbacks, can be found in [19].

Recently, an exciting new branch of TO has emerged, whose aim is not just to minimize the mass/stiffness ratio, but to take account of damage during service loading. In many engineering applications, damage or cracks may occur inevitably, and designing structures or materials able to continue working in damaged conditions may be crucial for safety in a variety of applications from the design of airframes to civil infrastructures [20]. Another important feature is energy dissipation during a static or impact load, where it might be desirable to authorize some damage/plasticity/cracks to dissipate energy and preserve other critical locations within the structure. Structures of this type are needed in a wide variety of fields including crashworthiness designs, seismic applications, blast protection, personal safety and sport equipment, among others [21]. Finally, resisting to fatigue by accurately designing the geometry is also critical for engineering performances. These different features can be obtained by optimizing the structural geometry, or the microstructure of a material itself, thanks to recent progresses in 3D printing [22–25]. Globally, these different applications can be handled by Topology Optimization to fracture resistance and are of industrial and technological critical importance, for applications in aircraft, automotive or biomechanics, among many others.

TO capable of taking crack propagation into account poses enormous challenges, as it can combine TO with fully nonlinear path-dependent simulations. This is in contrast with previous linear approaches. The first challenge is then the computational cost. The second issue is the need for a robust and efficient crack propagation simulation method, compatible with TO. Usually, TO requires fixed meshes for computational efficiency.

This makes classical FEM-based crack propagation methods with remeshing (see e.g. [26, 27]) not well adapted to TO. The eXtended Finite Element Method (XFEM) [28–31] offers the possibility to simulate cracks over fixed meshes. However, as being based on linear fracture mechanics, it can not handle crack initiation. In addition, the simulation of complex and multiple cracks in XFEM induces an enormous level of complexity, which limits its application in TO, even if there are a few examples, as presented in this review article. The phase field method to fracture, developed by Marigo, Bourdin, Francfort and Miehe [32–35] has many advantages for being easily combined with TO: first, it can operate on fixed meshes, as based on a regularized gradient damage model; then, it can easily handle initiation, propagation and merging of cracks with arbitrary geometries. For these reasons, its combination with TO, initiated in the seminal paper by Xia et al. [36], has opened the way to a large number of new methodologies for complex applications, from structural ones to material engineering.

In this article, we present an overview of these methods, combining TO and fracture propagation, but also all the previous and related approaches that have led to these techniques. Note that we focus on Topology Optimization, and do not review other optimization techniques like parameterized geometries (see e.g. [37, 38]) for fracture resistance, or related works like optimal control for cracks [39], even though these studies share common objectives with TO for fracture resistance. In section 2, we first review stress-based methods, which were the forerunners of crack resistance methods by producing optimal designs that prevents any initiation of damage or cracking. In section 3, works taking into account the presence of defects or cracks in structures, but following classical approaches aimed at minimizing compliance in an elastic framework are presented. In section 4, TO methods for fatigue damage are reviewed. In section 5, we then describe more recent approaches, including non-linear effects in the design of structures, such as plasticity and damage. Finally, we provide in section 6 a recent state-of-the-art of most recent TO design methods to fracture resistance, including an explicit description of crack propagation during loading, from initiation to failure of structures and materials. In particular, the design of two-phase

materials that are more resistant to cracking and can be manufactured by 3D printing is discussed. The article concludes with some challenges and promising avenues for the coming years in this field.

## 2 Stress-based approaches

Initially flourishing in compliance optimization, stress-based design is another prominent and active topic within the TO community. The first work on TO for eliminating stress concentration can be traced back to [40]. Another pioneering work, recognized in the field, employs the SIMP approach to address local stress constraints [41] by Duysinx and Bendsøe. Currently, stress-based design problems have extensively explored various TO approaches mentioned earlier. One commonly studied benchmark is the L-shape beam, which aims to eliminate stress concentrations in the right-angle area (a brief summary is provided in Fig. 1). The single-constrained stress-based optimizations can be categorized into two types: mass minimization under maximal stress constraints [42–53], or conversely, maximal stress minimization under volume constraints [54–60].

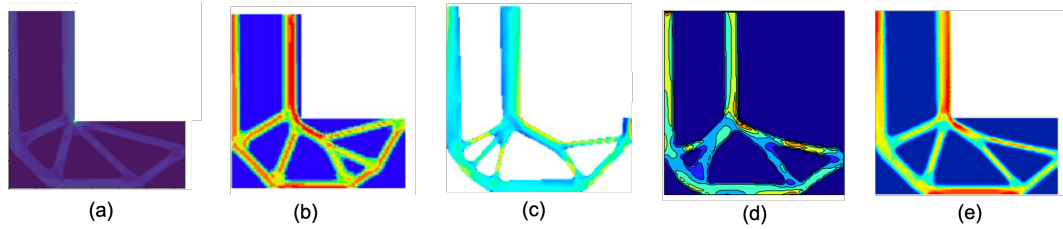
Stress-based TO poses greater challenges compared to compliance-based TO due to several factors: (1) stress is inherently a local characteristic, (2) it involves non-self-adjoint optimization, and (3) the imposition of numerous stress constraints is required. In order to address these challenges, aggregation functions such as  $p$ -norm [61] and Kreisselmeier-Steinhauser (KS) functions [40] have been widely adopted to represent the stress constraint corresponding to the number of finite elements or Gauss points. However, achieving a balance between accurately predicting the maximum local stress and employing a gradient-based robust algorithm for highly nonlinear stress optimization is crucial. This balance heavily relies on the choice of aggregation degree, such as the  $p$ -value in the  $p$ -norm function. Recently, the application of Heaviside projection-based aggregation [62] in stress-based TO [63] has enabled precise predictions of maximal stress while significantly reducing computational complexity. To tackle large-scale design problems, augmented Lagrangian approaches have been proposed [64–67], where solutions involving over 135 million elements and 505 million stress constraints have been

achieved. Similar strategies have been employed in the context of reliability-based topology optimization (RBTO) [68] and in considering manufacturing uncertainties [69], respectively. Additionally, a comparison of local and global stress constraint strategies in TO is presented in [70].

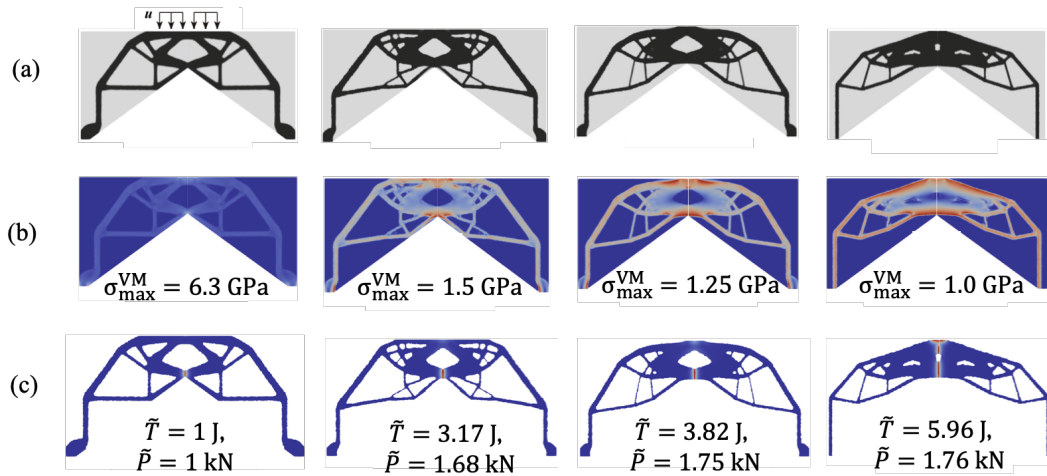
The stress-based design approach was initially combined with classical compliance topology optimization in [40], where the simultaneous minimization of stress and compliance was carried out under a mass constraint. Subsequent studies, such as [74–78] and [79, 80], focused on formulating compliance minimization with stress and mass constraints, as well as achieving minimum weight while considering both compliance and stress constraints, respectively. In [81], Boissier et al. used a Level-set TO to solve cyclic elastoplastic problems based on the Melans theorem, in which only the existence of certain residual stress fields based on elastic solutions is required to guarantee asymptotic elastoplastic behavior (shake-down). In [74], the optimization of elastoplastic and incompressible media was made possible by employing a mixed finite-element discretization scheme. Recently, the same optimization formulations were solved using the BESO method for linearly elastic materials in [82, 83] and the LSM for shell structures in [84]. Notably, [71] by Da and Chen represents the first work that systematically analyzes the effects of stress-based designs on fracture resistance properties (see Fig. 2). The authors found that minimizing the critical maximum stress<sup>1</sup> can delay the onset of cracking, and when combined with compliance optimization, it leads to significant enhancements in both peak load and toughness. Furthermore, the authors employed a data-driven strategy in [85] to minimize multiscale homogenized stress, indirectly enhancing the toughness of heterogeneous structures. The study emphasizes the significant size effects observed in multiscale stress-oriented irregular architected materials (see Fig. 3).

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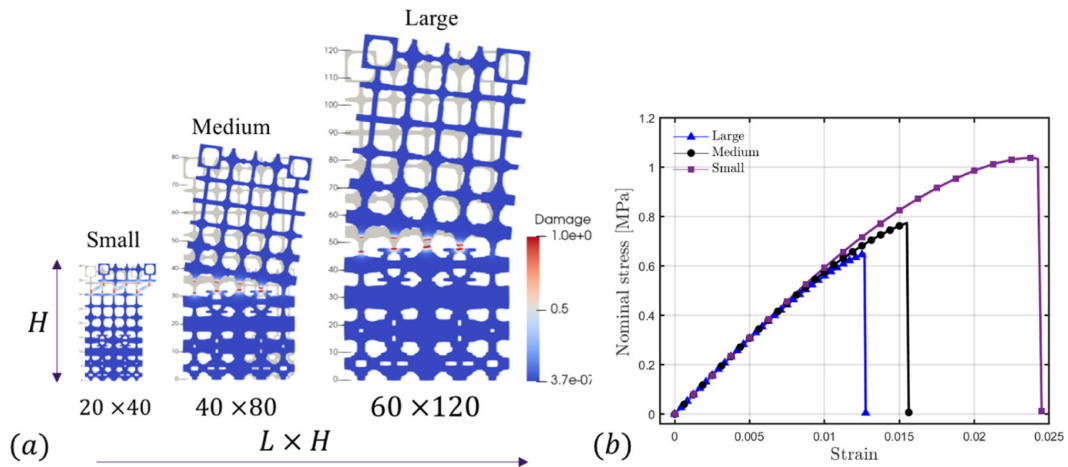
<sup>1</sup>By critical, we mean the stress concentration directly contributing to crack initiation.



**Fig. 1** Stress distribution of the optimal L-shaped beams achieved using various stress-based TO methods: (a) compliance TO with stress concentration [71], and (b-e) the following approaches: (b) SIMP [61], (c) BESO [60], (d) LSM [72], and (e) MMC [73]. Reproduced figures used with permission.



**Fig. 2** Effects of stress-optimized designs on fracture resistance properties [71]: (a) diverse optimized topologies for various scenarios; (b) stress distribution highlighting the maximum von Mises stress; (c) fracture patterns displaying normalized toughness ( $\tilde{T}$ ) and peak load ( $\tilde{P}$ ) values. Reproduced from [71] with permission.



**Fig. 3** Size effects of the global heterostructure: (a) final crack patterns of the data-driven designed structures; (b) corresponding nominal stress-strain curves. Reproduced from [85] with permission.

### 3 Linear elastic structures containing virtual or fixed pre-existing cracks

One of the first reported work introducing the notion of crack in TO is by Challis et al. [86]. In their study, an objective function defining a virtual crack extension was proposed on the boundary nodes of a structure discretized by Finite Elements. The framework only involved linear elastic structures.

A series of other works have investigated the TO of structures embedding pre-existing cracks or defects within a structure. In these problems, a linear elastic structure was usually considered, and the TO was carried out with respect to compliance volume, or toughness, taking into account the presence of fixed cracks. In an early work [87], Jansen et al. defined a TO framework for minimizing compliance of structures such that they were less sensitive to defects. For this purpose, several “worst case” scenarios were defined, where material was totally damaged in a square patch. The compliance was then minimized for a given number of damage scenarios. As a result, the optimized structure was assumed to be insensitive to cracks in the patch zones.

In [88], Shobeiri investigated TO of linear elastic structures containing fixed cracks by combining the Element Free Galerkin Method (EFG) [89] and BESO. Differences between EFG and FEM were reviewed in this context.

An innovative contribution was proposed by Buehler’s group in [90], where a TO framework was introduced to optimize for the first time the toughness of a bi-material sample. In their work, the TO of a notched plate made of soft and stiff phases distributed on a regular grid was carried out to improve the toughness under mode-I failure. A greedy algorithm was proposed to obtain the topology of the second phase, and finite elements were used to evaluate the toughness using linear analysis. The toughness was here defined as the area under the force/displacement curve at failure strain. In a follow-up work [91], the same authors used the optimized topologies to 3D-printed samples made of soft and hard phases (see Figs. 4 and 5). In these studies, no crack propagation was considered in the simulations and thus in the TO process, but the samples were

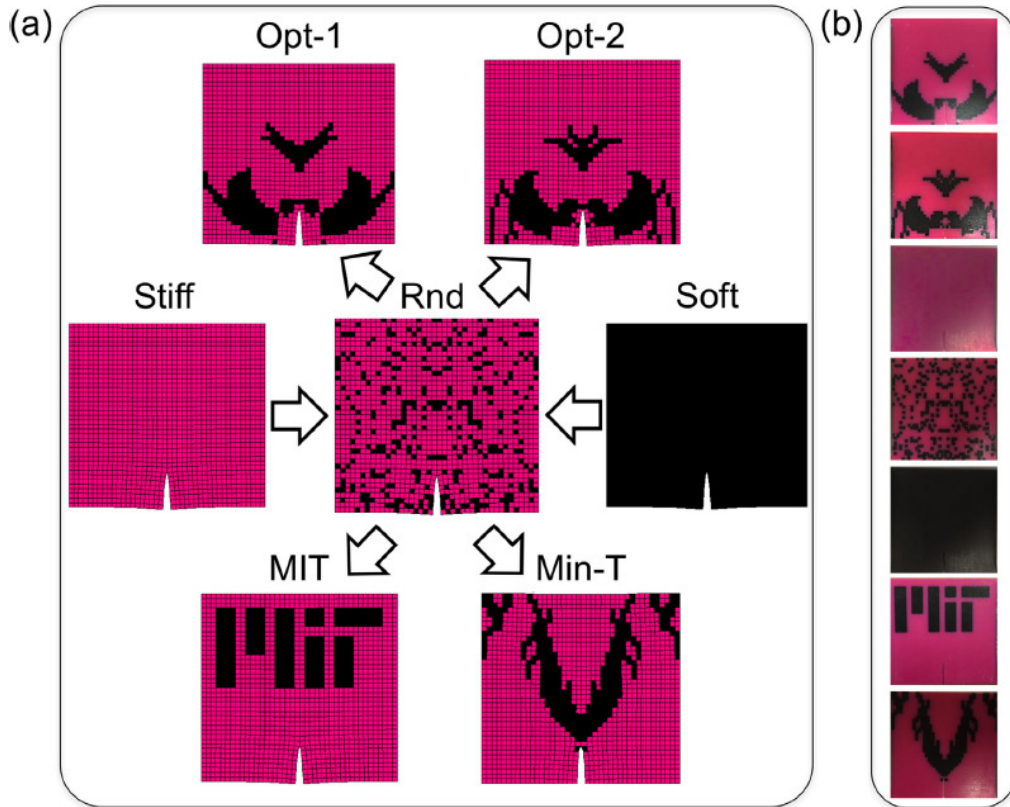
experimentally cracked to evaluate the toughness improvements (see Fig. 5). It was shown that a redistribution of soft and stiff phases could lead to a clear improvement of toughness and strength as compared to other benchmark designs.

In [92], Kang et al. evaluated the J-integral around predefined cracks at specific locations and used it in the objective function to optimize the topology of elastic structures. In [93] Banh and Lee used SIMP and a multi-material TO to minimize compliance in presence of fixed cracks, where the discontinuities and singularities around the crack tip were modelled by XFEM [28, 29]. The same authors extended this approach to Functionally Graded Materials (FGM) in [94]. A similar procedure was developed by combining XFEM and BESO in [95] (see Fig. 6), as well as in [96] to define the topology of a second material playing the role of patches near existing cracks to minimize the energy release rate within a BESO framework. Mechanical ice protection systems was investigated in [97] using topology optimization to enhance ice shedding through fracture.

Finally, peridynamics [98] was investigated for compliance optimization of linearly elastic cracked structures in [99–102]. A comparison between peridynamics and FEM in this context was discussed in [101]. In [102], Kendibilir et al. fabricated optimized structures by 3D printing and tested it experimentally.

## 4 Fatigue

Fatigue resistance TO is primarily stress-based and can be enhanced by controlling the stress level. It typically requires ensuring that the stress level of each finite element remains within the safe region, i.e., below the yield stress, after a certain number of stress cycles. The adoption of stress aggregation functions, such as the  $p$ -norm, allows constraining the maximum accumulated “damage” to maintain it at a tolerable level even after a significant number of stress cycles [103]. Fatigue resistance in TO can be categorized based on the number of cycles as low-cycle fatigue (LCF) or high-cycle fatigue (HCF), with a dividing line typically set around  $10^4$  cycles [104]. Due to the complexity of LCF, which is primarily governed by plasticity, the majority of research in fatigue TO has focused on HCF. One notable example of TO work addressing LCF design is found in [105]



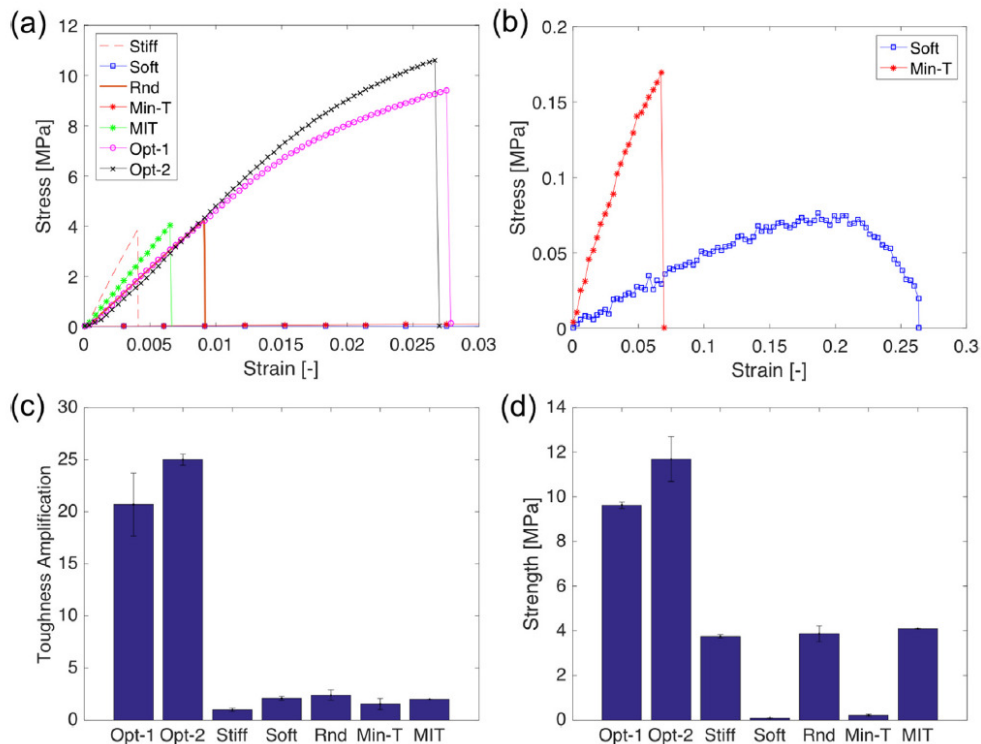
**Fig. 4** Bi-material notched plate (a) Several designs with different distributions of the soft and stiff phases. (b) 3D printed samples. Reproduced from [91] with permission.

by Desmorat and Desmorat. In their study, fatigue life was maximized by optimizing the topology of a 3D double-clamped cantilever beam, considering cyclic plasticity in conjunction with the Lemaitre damage law [106, 107].

In the context of HCF designs, early studies such as [108] employed three different HCF criteria (Sines [109], Crossland [110], and Dang Van [111]) within a parametric structural optimization framework, solved using a gradient-free evolutionary algorithm. This work was followed by a TO framework by the same author in [112], which demonstrated that TO can effectively reduce the mass of a structure while maintaining its durability at an established level. In the same year, Sherif et al. [113] investigated dynamic response TO, where fatigue played a crucial role in determining the optimization process, and proposed a criterion for meaningful equivalent static loads (ESL) [114]. Subsequently, Yoon's group comprehensively incorporated dynamic fatigue and

static failure criteria into TO in their works [115–117], utilizing Miner's cumulative damage rule to calculate the total damage at each spatial element.

In [118] and [119], the same cumulative damage rule was employed, with the principal stress and Sines damage criterion being constrained in TO, respectively. The use of the clustered approach and Rainflow-counting method in these works simplifies the optimization problem, making it as convenient as the classical von Mises stress-based TO framework. However, both studies show that the optimized topologies obtained through fatigue-based TO exhibit similarities with stress-based designs. In contrast, [120] adopts a continuous time approach for fatigue, where the evolution of damage is governed by a system of ordinary differential equations. This approach does not rely on cycle-counting techniques such as Rainflow-counting and integrates fatigue damage over the entire stress history. Consequently, it can handle more general load histories, including those with



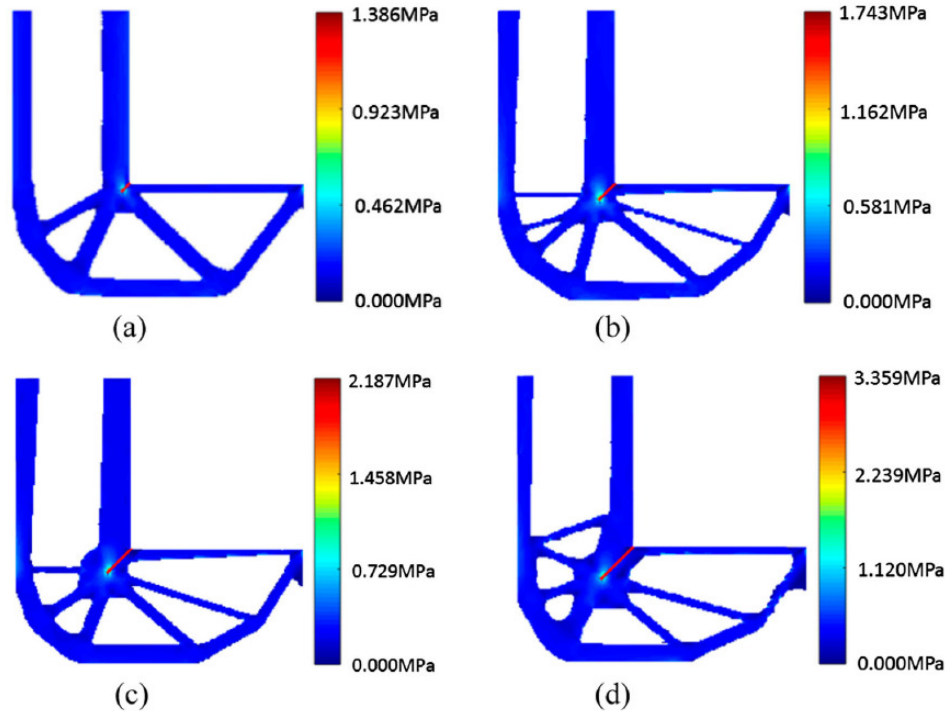
**Fig. 5** Stress-strain relations. (a) Both optimized materials fail at a higher stress and strain compared to all benchmark samples. (b) Stress-strain relations for the softer materials that exhibit higher strain and lower failure stress. (c) Toughness amplification. The two optimized geometries have on average a more than 20-fold increase in toughness modulus compared to the stiff homogenous sample. (d) In terms of strength, optimized geometries (see Fig. 4) have on average a more than two-fold increase in strength compared to stiff and random geometries. Reproduced from [91] with permission.

non-proportional loads. The computational acceleration of the proposed framework is addressed in [121], which presents HCF-constrained TO for a 3D bracket subjected to periodic load history. Furthermore, the investigation of fatigue-based TO under non-proportional loads has been recently explored in [122], where different objective functions, including maximizing fatigue life under volume constraints, were studied.

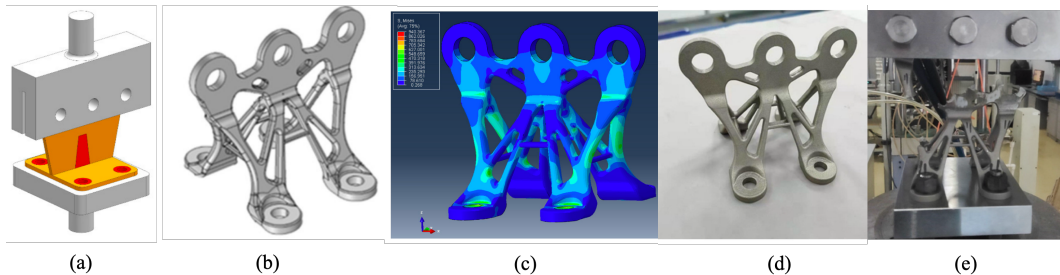
Zhao et al. [123] addressed the same finite-life local fatigue under random vibrations using a density-based SIMP-classified method, similar to the other reviewed works. This problem was recently solved using the BESO method [124] and unstructured polygonal meshes [125]. In contrast, Collet et al. [126] proposed infinite-life fatigue TO, aiming to minimize mass while subject to compliance and simplified nominal stress constraints. They adopted a modified Goodman failure criterion based on the Sines method.

Due to the prevalence of fatigue failure as a leading cause of rupture in fastened parts, fatigue TO has gained significant attention in real-world engineering applications. This is particularly evident with the rise of additive manufacturing, which has led to numerous engineering examples and experimental investigations (see Fig. 7). A recent study by Olesen et al. [131] explored the simultaneous optimization of topology and print orientation for transversely isotropic fatigue. Suresh et al. [132] investigated TO for additive manufactured components and transversely isotropic materials, focusing on HCF as a constraint. Trudel et al. [133] proposed penalization techniques for fatigue TO of structures incorporating embedded functionally graded lattice materials. Correspondingly, several studies have examined fatigue TO for real engineering and medical parts, such as the Helicopter Tail Rotor Pitch Arm [128], Aerospace Bracket [129], and Temporomandibular Joint Prosthesis [130] (see





**Fig. 6** L-bracket optimization: von Mises stress contour in the optimized geometry for several initial cracks with different lengths. Reproduced from [95] with permission.

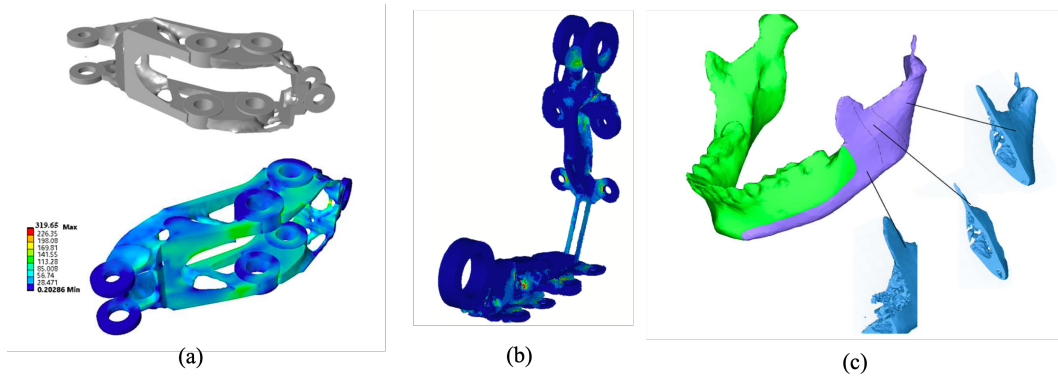


**Fig. 7** Fatigue TO and experiments of a 3D-Printed metal aerospace bracket [127]: (a) assembly of the original bracket and the fatigue testing platform; (b) topology of the optimized bracket; (c) stress analysis of the new bracket; (d) 3D printed sample; and (e) experimental fatigue test. Reproduced from [127] with permission.

Fig. 8). Additionally, Dagkolu et al. [134] reported the design of a fatigue-critical aerospace part using TO, which was manufactured using laser power bed fusion (L-PBF). Lastly, Liu et al. [135] achieved enhanced fatigue characteristics in a topology-optimized porous titanium structure produced via selective laser melting.

## 5 Path-dependent approaches: plasticity and damage

Stress-based approaches have limitations to correctly address situations where damage has already formed, and to correctly measure failure of a structure. Therefore, the stress-based design procedures may produce structures that



**Fig. 8** Fatigue TO of actual engineering components of (a) Helicopter Tail Rotor Pitch Arm [128], (b) aerospace bracket [129]; and (c) Temporomandibular joint prosthesis [130]. Reproduced from [128–130] with permission.

lack robustness in such scenarios [20]. More specifically, for materials having highly nonlinear behavior, such as elastoplastic and strain-softening, once a threshold for stress or strain is reached, the material loses part of its stiffness but may experience large deformations before actually failing. For these materials, failure cannot easily be expressed in terms of a maximal stress or strain. By constraining the stress to be lower than the yield stress, the optimized designs will be structurally safe but overly conservative since the behavior beyond yielding or damage initiation is not considered [136]. To overcome these limitations, authors have introduced TO approaches including fully nonlinear and path-dependent simulations. Early approaches have first included plasticity in TO approaches, then elastic damage models, and later models coupling damage and plasticity.

## 5.1 Plasticity

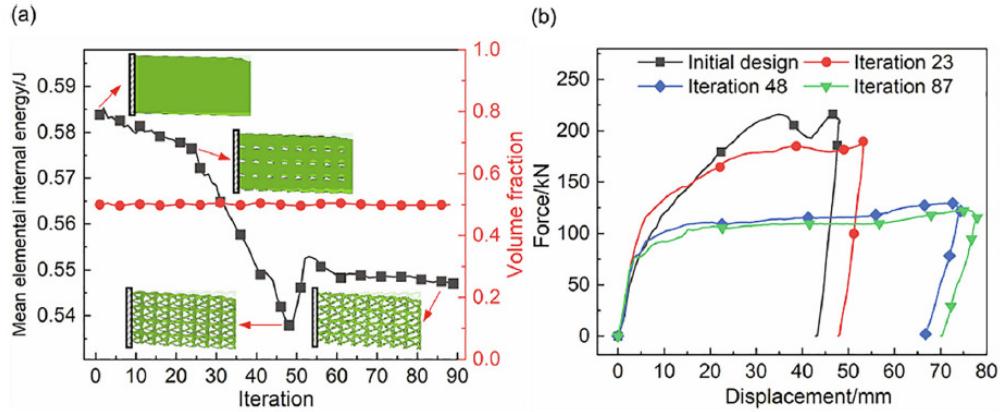
Plasticity can be the onset of failure in ductile materials. A first contribution to TO in path-dependent problems with plasticity was proposed by Maute et al. [137, 138], where an elastoplastic structure was considered in the simulations. In their work, the structural ductility, defined as the integral of the strain energy over a given range of a prescribed displacement, was maximized under mass constraint. In [139], Kato et al. proposed a SIMP TO in composites considering elastoplastic deformations to maximize the energy absorption capacity (defined as the external work) of a structure under a prescribed material volume and a whole incremental simulation. Analytical sensitivities were derived for the path-dependent problem.

In [140], Nakshatrala and Tortorelli introduced the TO of dynamically loaded structures with rate-independent elastoplastic material behavior with applications to impact mitigation. An extension of TO for elastoplastic structures at finite strains was performed by Wallin et al. in [141]. Hencky’s law of plasticity with regularization was investigated within the Level-set TO method in [142].

Recently, Jia et al. [143], investigated material design by performing TO in cellular periodic structures considering plasticity and dynamics, showing that energy absorption could be effectively improved as compared to classical microstructures such as honeycombs (see Fig. 9).

## 5.2 Damage

Pioneer works combining topology optimization and damage may be attributed to Achtziger and Bendsøe [144], who designed truss structures with a damage model. In a next study, Bendsøe and Diaz [145] used an elastic-damage model for designing structures with damage constraints. In [37], Desmorat and Desmorat combined TO with continuum damage to maximize the lifetime of structures subject to fatigue damage. In [146], Amir included discrete steel reinforcements embedded into a concrete continuum, which was subject to damage. In [147] Amir and Sigmund optimized reinforcements in concrete structures, where the concrete was described by a gradient-enhanced damage model with strain softening, and reinforcements modeled by elastic bars embedded in the matrix. In this work,



**Fig. 9** Iteration of cellular structure with  $8 \times 4$  periodic unit cells: (a) iterative histories of mean energy absorption and volume fraction constraint; (b) force-displacement curves of different iterative steps. Reproduced from [143] with permission.

final compliance was minimized but after solving a fully nonlinear incremental path-dependent problem (see Fig. 10).

In [148], James and Waisman used TO to generate minimum-weight structural layouts, subject to a compliance constraint. Failure mitigation was achieved by constraining the maximum local damage. The damage was modeled using a continuum damage approach in which the nonlocal damage evolution law was coupled to a finite element model. In a follow-up study [20], the same authors performed the TO of structures able to sustain material damage and subjected to multiple loading cases, using a coupled nonlinear brittle damage model. A nonlocal damage model was developed by Noël et al. in [149] in the framework of a level-set TO. More recently, Barbier et al. [136] considered minimization of volume in a structure by solving an elastic problem with nonlinear damage model [150] under a maximum loading capacity, including a fully nonlinear strategy and a simplified strategy, where damage was computed into the structure in only one step.

### 5.3 Plasticity and damage

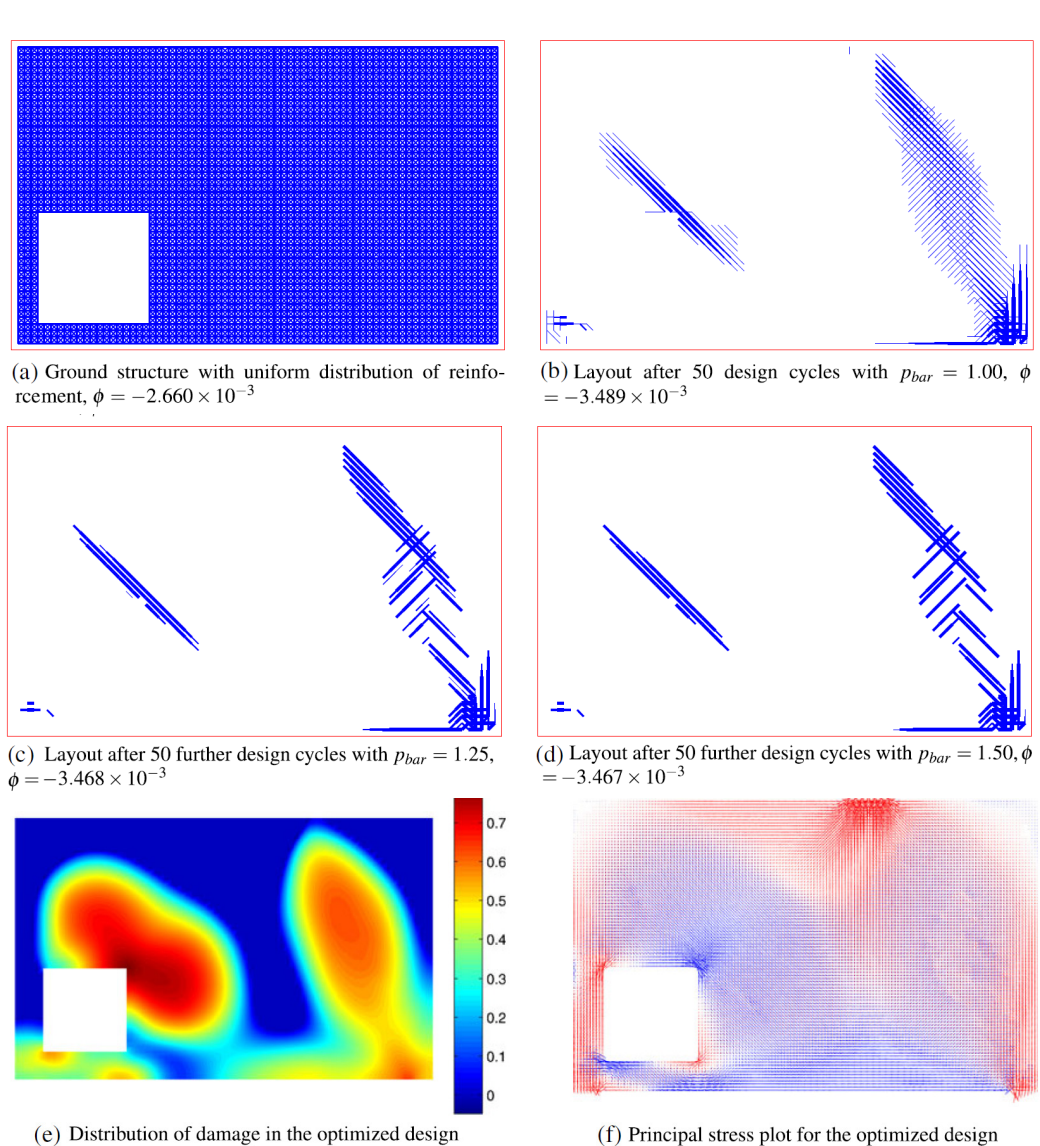
The aforementioned approaches only included damage in the nonlinear analysis. Other works coupled damage and plasticity in TO path-dependent strategies to maximize the load bearing capacity of elastoplastic structures with damage.

In [151] Alberdi and Khandelwal optimized the volume of elastoplastic-damageable structures

under maximum plastic work and maximum damage constraints. In [152], Li and Khandelwal developed the optimization of structures resistant to ductile fracture by using elastoplastic-damage path-dependent simulations for minimizing the structural volume under maximum plastic work and maximum damage constraints. Similar approaches were developed by the same authors in [153] where the plastic work was maximized under damage and volume constraints, and in [154] using a nonlocal damage model. More recently, Zhang and Khandelwal [155] extended such strategy to a finite strains Gurson-Tvergaard-Needleman plasticity model within a SIMP TO framework, where the plastic work was maximized under damage and volume constraints (see Fig. 11)

## 6 Path-dependent approaches: crack propagation

A new step in the development of TO to fracture resistance was accomplished by incorporating an explicit description of crack propagation in path-dependent TO approaches. To our best knowledge, this has been firstly proposed in Zhang et al. [156], where an XFEM fracture propagation was combined with a BESO TO method. The emergence of such approach has been accelerated by the recent development of the phase field method to fracture (see e.g. [32–35, 157, 158]), which has considerably simplified the analysis of fracture propagation in engineering problems. The first combination of the phase field method to fracture with TO was



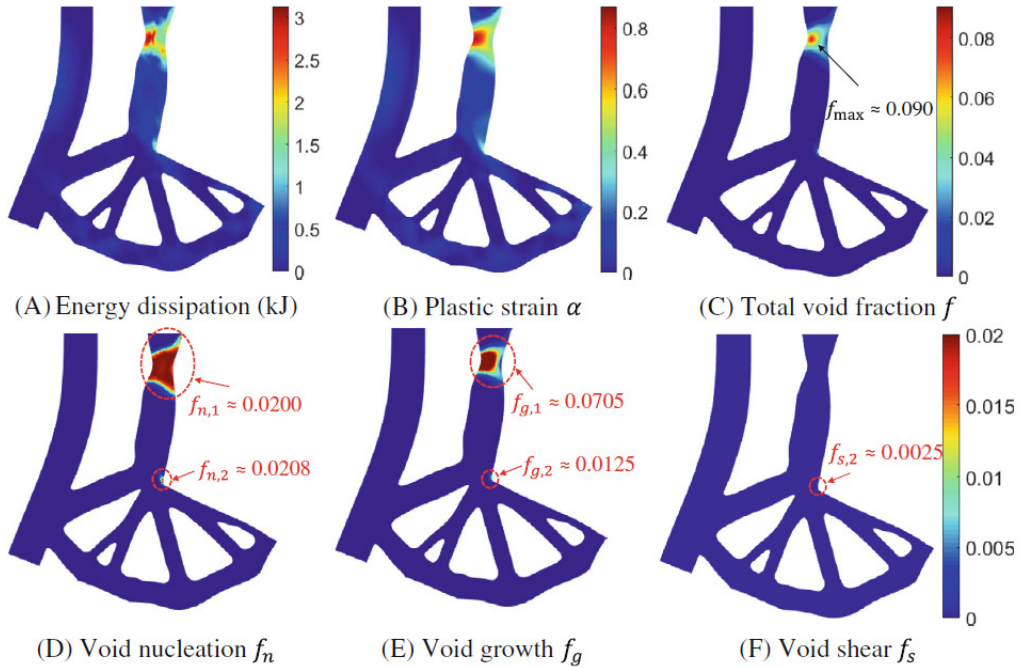
**Fig. 10** Optimized reinforcement layouts and response of a 2-D structure taking into account damage. Reproduced from [147] with permission.

achieved by Xia et al. in [36] and Da et al. in [159] in the framework of bi-materials, and then followed by several groups both in the contexts of single and bi-material structures with different variants regarding the definition of the objective function and the TO technique. The different methods are described in the following.

## 6.1 Single material structures

In [160], Russ and Waisman developed a SIMP TO framework with path-dependent phase field

simulations. In their work, the total mass of the structure was minimized under maximum compliance and fracture energy constraints during fracture propagation simulations. A careful analysis of the effects of phase field model features, like degradation function and strain splits was carried out to investigate their effects on the final topologies. It was evidenced that including the fracture energy as a constraint could improve the fracture resistance as compared to including the compliance



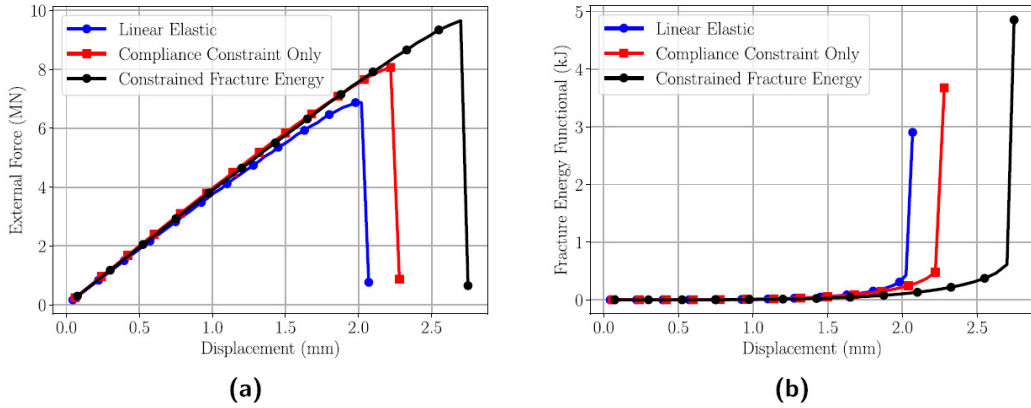
**Fig. 11** Optimized L-bracket taking into account path-dependent finite strains Gurson-Tvergaard-Needleman plasticity model and damage. Reproduced from [155] with permission.

constraint only (see Fig. 12). In [161], a threshold formulation within phase field was investigated by the same authors, and a different objective function combining mass and external work under constraint of maximal cumulated external work was analyzed.

These works have been followed by similar studies with different variants in the choice of TO methods or definition of the optimization problem. In [162], Wu et al. used a level-set TO combined with phase field fracture and a  $p$ -norm function formulation to aggregate the phase field variables into a single constraint. The authors evidenced the practical interest of such approaches in dental bridge applications. In [163], Desai et al. also developed a Level-set TO combined with fracture simulations to minimize total external work during fracture simulations of structures and conducted advanced 3D applications (see Fig. 13). In [164], Jia et al. combined a SIMP TO including phase field fracture and used a multi-objective function that allowed optimizing in a weighted manner: (i) the initial stiffness of the structure; (ii) the first instance at which fracture nucleates and (iii) the energy dissipated by fracture propagation once fracture has occurred (see Fig. 14).

All the above-mentioned works only considered static, quasi-brittle structures. In [165], Li et al. developed a ductile fracture behavior combined within a BESO TO formulation. Another novelty of this work was the introduction of the anisotropy in the fracture behavior, to account for orientation in 3D printed fabrication process (see Figs. 15-16). In a recent paper, [166], Noii et al. also investigated TO with ductile fracture within a Level-set TO. In their work, the total work was maximized while accounting from structural damage and mass constraints and the level-set surface used within TO was updated by a reaction-diffusion algorithm.

Finally, another recent and promising advance is the consideration of dynamic fracture within TO. The first related study and the only one so far to our best knowledge has been conducted by Wu et al. in [168], where SIMP TO was combined with dynamic phase field propagation analysis. In this work, fracture energy was minimized during a given time load after impact on the structure and with volume constraints. Introducing dynamic effects allowed to take into account the effects of loading rates (see Fig. 17).



**Fig. 12** Force-displacement (a) and fracture energy functional (b) curves using the principal split of the elastic energy and the quadratic degradation function for three topologies. Reproduced from [160] with permission.

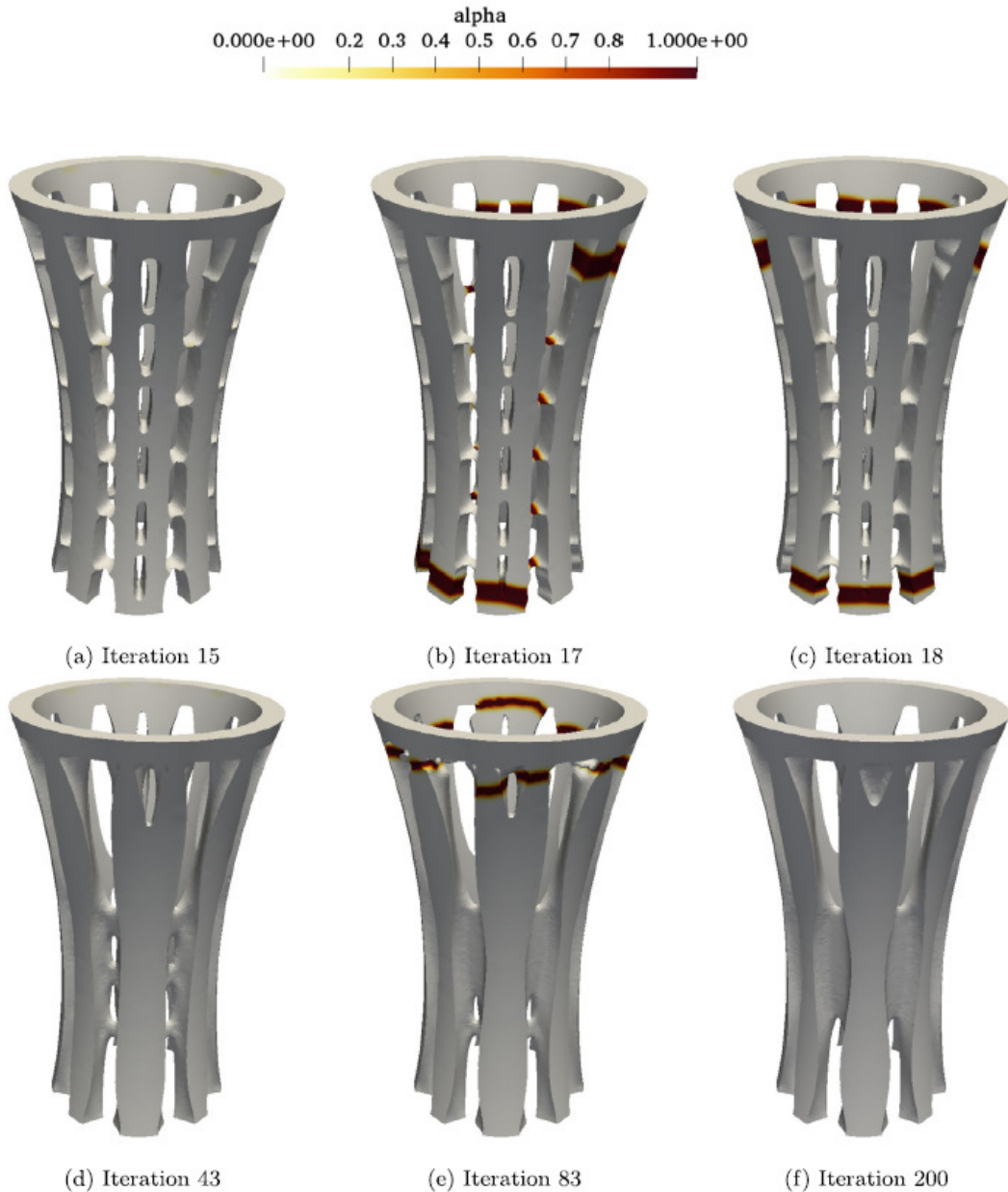
## 6.2 Heterogeneous structures and materials

The structural design discussed above is primarily derived from the pioneering work by Xia et al. [36] and Da et al. [159], with a specific emphasis on two-phase quasi-brittle composite structures. In their study [36], the authors employed TO to optimize the distribution of the stiffer phase in order to maximize the absorbed energy throughout the fracture process, encompassing damage, crack initiation, propagation, and ultimate failure. Furthermore, they perform a comprehensive sensitivity analysis of the objective function with respect to the topological design variable. The paper showcased numerical examples in both 2D and 3D settings, illustrating the enhanced fracture resistance achieved when compared to classical linearly elastic compliance-based TO approaches.

Da et al. [159] presented a significant advancement by addressing the widespread issue of interfacial damage and its implications. The authors introduced a novel approach to optimize the topology of reinforced inclusion phases by modifying the geometry of the interface. Their objective function was designed to maximize toughness considering entire reaction force and load-point displacement curves, with a constraint on the inclusion volume. The study investigated scenarios with and without initial cracks, shedding light on the interfacial effects. By employing a highly nonlinear cohesive zone model [169, 170], the researchers observed that cracks tend to initiate at the optimized interface in the absence of

predetermined or initial cracks. Through the optimization process, the study reveals the emergence of concave interfaces, effectively preventing inclusion bypass and promoting crack nucleation on the opposite side of the inclusion. Moreover, the concave interfaces exhibit twisting behavior and elongate the path of interfacial crack propagation, consequently demanding elevated fracture energies to fracture the component. These insights have a direct impact on fracture resistance, offering an effective mechanism for toughening through crack bridging and twisting.

Li et al. [167] and Wu et al. [171] have both employed a similar framework, utilizing different TO methods such as SIMP and LSM, respectively. Li et al. [167] compared the BESO and SIMP methods by analyzing various initial guess designs. In their study, Wu et al. presented a practical example of an all-ceramic dental bridge, demonstrating the improved fracture toughness achieved by combining LSM with phase-field fracture modelling. Expanding upon this framework, Da and Yvonnet [172] incorporated periodic composite structures and multiple objectives into their work. They enforced the periodicity of the unit cell geometry while performing TO for a given load applied across the entire structure. The authors investigated size effects, which exhibited similar trends to classical fracture experiments in quasi-brittle structures (refer to Fig. 18). Additionally, they explored enhancing the fracture resistance of a unit cell associated with a material under multiple loads, resulting in a complex optimized

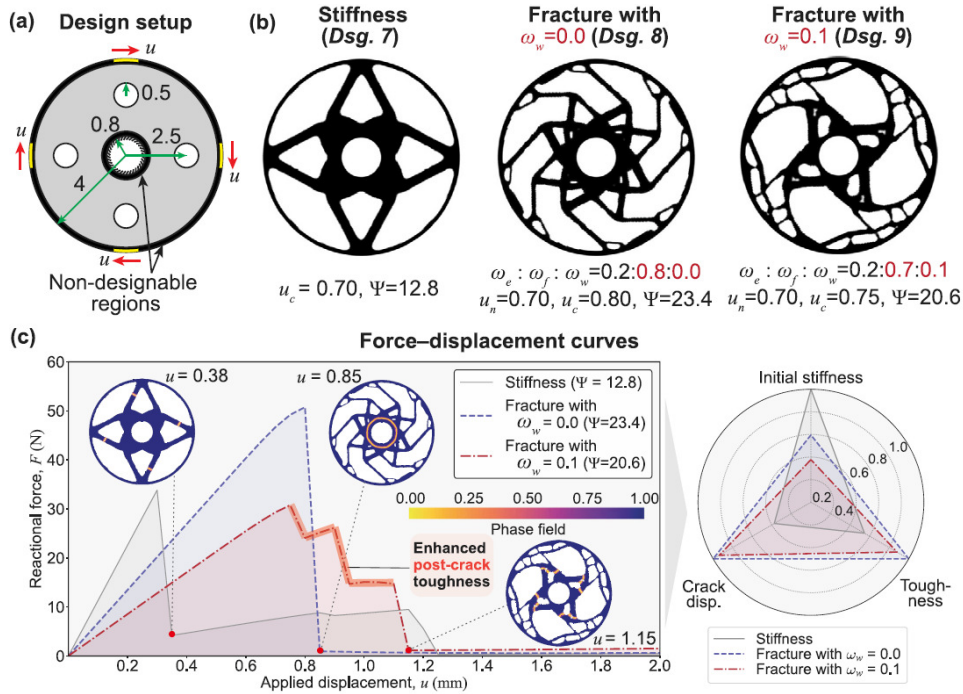


**Fig. 13** Damage variable at the final time plotted for several optimization iterations (black regions indicate cracks). Reproduced from [163] with permission.

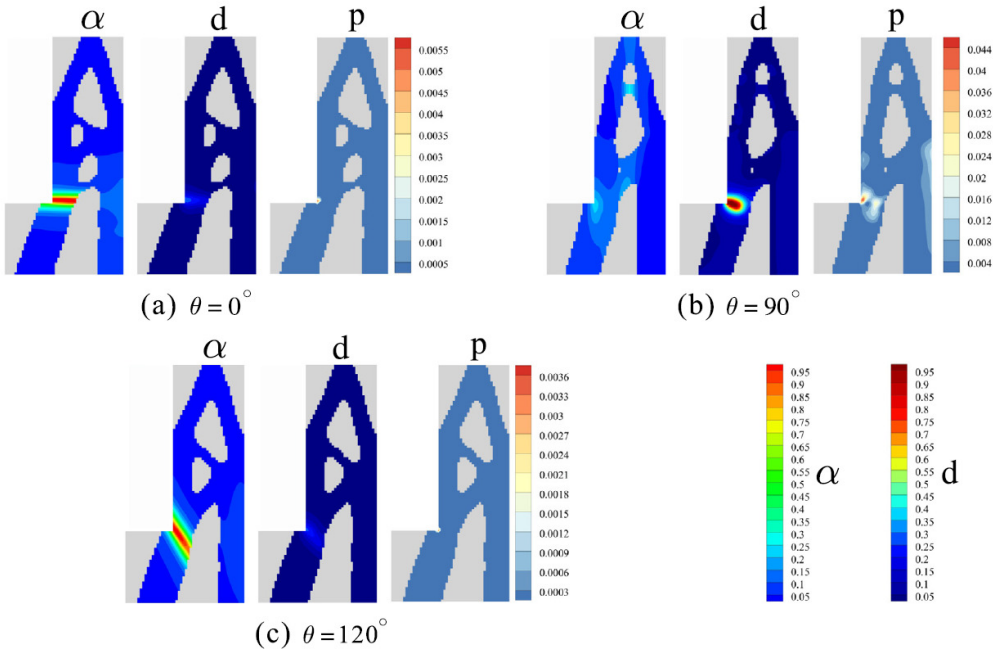
microstructure (refer to Fig. 19). The aforementioned studies [36, 159, 172] comprise the second segment of Da’s book [173] in TO of heterogeneous materials and structures.

In [174], a departure from linear analysis in assessing the toughness of bio-inspired bi-material samples, as seen in [91], was made by introducing phase field modeling. The primary focus of the

study is the analysis of the “brick-and-mortar” biological composites and its interlock and non-interlock architectural associations. Through this investigation, the researchers identified two distinct fracture stages that align closely with experimental observations [175]. To further enhance fracture performance, the phase field-based TO framework aiming to maximize toughness while

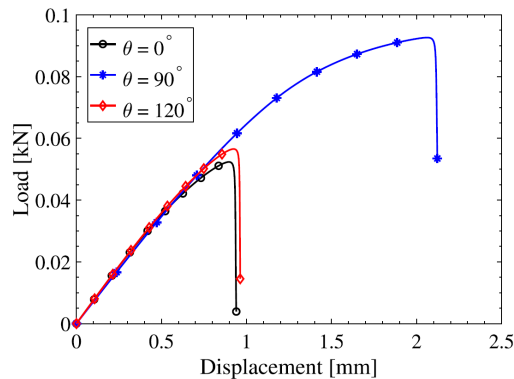


**Fig. 14** Design setup and results for the optimization of a disk under rotational displacement. (a) Design domain and boundary conditions (all length units are in mm). (b) Optimized designs for two different choices of weighting factors defining importance of initial stiffness, onset of fracture and energy dissipation. (c) Force-displacement curves for the optimized designs and corresponding radar chart comparing their normalized structural performance. The classical stiffness design obtained from conventional stiffness maximization is included for reference. Reproduced from [164] with permission.



**Fig. 15** Optimized designs of an L-shape structure with anisotropic ductile fracture behavior arising from 3D-printing process. The values  $\alpha$ ,  $d$  and  $p$  denote interfacial damage, bulk damage and equivalent plastic strain, respectively. The topologies and fields are depicted for 3D printing orientations  $\theta$ : (a)  $\theta = 0^\circ$ ; (b)  $\theta = 90^\circ$ ; (c)  $\theta = 120^\circ$ . Reproduced from [165] with permission.





**Fig. 16** Load-displacement curves for an L-shaped bracket (optimized design), showing the effects of 3D printing orientation angles  $\theta$ , and the transition from quasi-brittle to ductile fracture regimes. Reproduced from [167] with permission.

accounting for the mass constraint of the soft phase was proposed (refer to Figs. 20 (a-f)). Through strategic modifications to the constitutive material distribution, the authors have successfully eliminated the significant stress concentrations located ahead of the crack tip (refer to Figs. 20 (g) and (h)). As a result, the energy dissipation capability is significantly enhanced, leading to a notable improvement in fracture performance.

In a recent study by Da [176], the author investigated similar soft and stiff bio-inspired bi-materials and compared them with interpenetrating phase composites, focusing on more advanced 3D cases. The research included model reduction techniques applied to 3D fracture resistance design, demonstrating significant improvements in critical fracture resistance properties while achieving computational efficiency in inverse design. Moreover, the author and co-workers successfully fabricated bi-material topology-optimized structures using 3D printing. Experimental tests, such as the model-I test conducted by Da et al. [177], revealed that the toughness of the bi-material structures in brittle fracture can be enhanced by more than 100 times compared to pure homogeneous structures with a 100% volume fraction of constitutive stiff material. Further investigations on design optimization of inclusion phases in bi-material composites against failure can be found in the works of Singh et al. [178–180]. These studies focus on achieving delamination

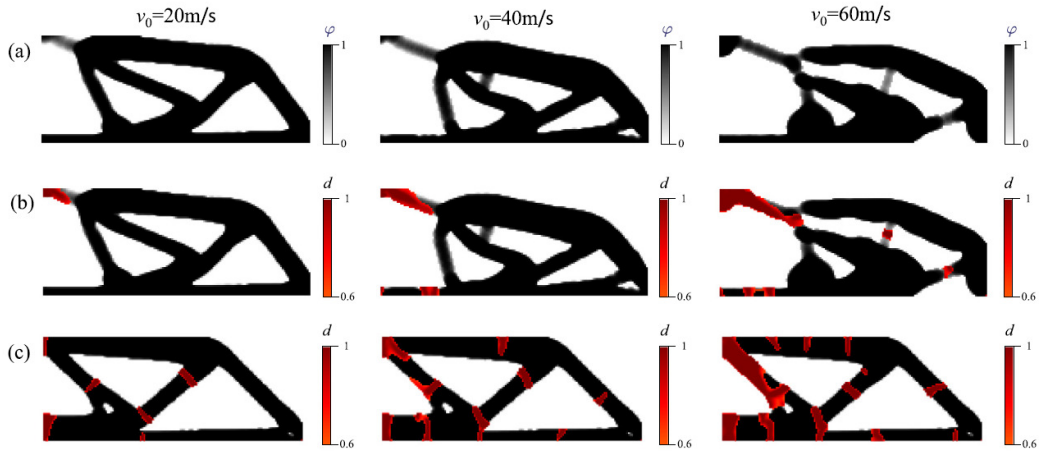
resistance in composite structures through variations in the size, rotation, and shifting positions of the inclusions within the context of periodic microstructure optimization. The similar crack bridging mechanism as in [159] offers valuable insights into diverse toughening mechanisms, encompassing crack nucleation, growth, and ultimate structural failure.

## 7 Conclusion and future challenges

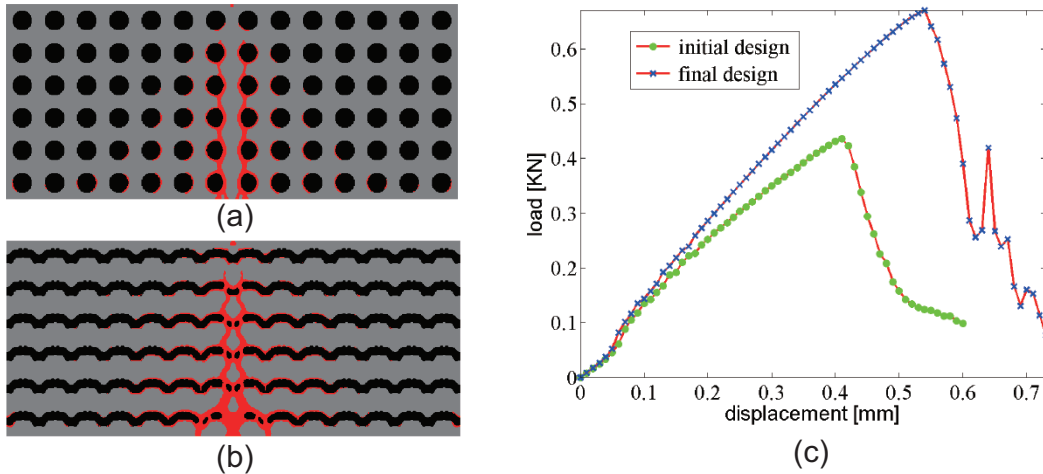
In this article, we have presented the state of the art of Topology Optimization (TO) for fracture resistance, including: (i) stress-based approaches, (ii) TO for linear structures containing initial non-evolving cracks, (iii) TO for fatigue resistance, (iv) TO taking into account path-dependent nonlinear damage and plasticity and finally (v) TO incorporating crack propagation in structures and materials.

Stress-based TO methods offer valuable insights for mitigating stress concentrations in linear structures, which is essential for ensuring structural integrity. However, they often have limitations when it comes to predicting plasticity, damage, or the behavior of cracks in scenarios where they cannot be entirely avoided. Fatigue TO approaches, on the other hand, predominantly rely on stress-based methodologies and are particularly useful for ensuring that stresses remain within safe limits over a specified number of load cycles. Nevertheless, there have been advancements within this framework to account for damage and plasticity. TO models incorporating pre-existing cracks primarily revolve around compliance optimization, wherein existing cracks are modeled as discontinuities in the structural design. These methods are well-suited for addressing structures with pre-defined flaws, offering insights into optimizing their performance.

TO techniques explicitly describing the development of plasticity, damage or crack propagation in load path scenarios add a strong layer of complexity, as they require the solution of a path-dependent nonlinear problem (usually with finite elements) at each iteration of the TO scheme. Various commonly used TO methodologies have been employed in these endeavors. The last-mentioned techniques represent the most recent contributions



**Fig. 17** Optimized structures for dynamic fracture resistance. Columns correspond to different loading rates; (a) Final design taking into account Dynamic fracture; (b) Final crack pattern; (c) design obtained by static analysis for comparison. Reproduced from [168] with permission.



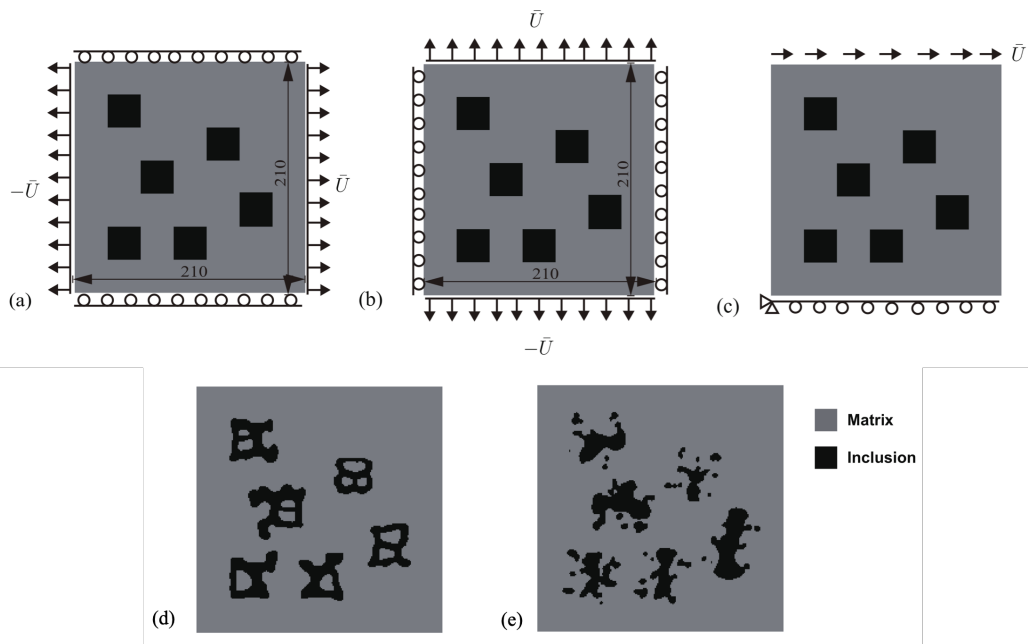
**Fig. 18** TO of a bi-material structure; black phases denote stiff inclusions, grey phase denote brittle matrix: (a) fracture (in red) in 3-point bending in the initial design; (b) fracture in the optimized design; (c) comparison between the response of initial and optimized designs. Reproduced from [172] with permission.

to TO for fracture resistance, and the various related studies have shown promising gains in terms of fracture toughness relative to mass, compared with previous design techniques. In addition, they also make it possible to develop new features, such as the ability to maintain structural integrity or to dissipate energy in the most efficient way in the case of unavoidable damage situations.

However, the development of these techniques for practical engineering applications still faces a number of technical and scientific challenges. The first of these is computational cost. Unlike

linear approaches, each iteration of the TO process usually involves a full non-linear simulation. Advanced computational methods are therefore needed to drastically reduce these costs for engineering applications. In this respect, model reduction or machine learning can be promising ways of solving this problem.

Another important future challenge will be to develop multi-scale approaches to more accurately take into account the effects of microstructures or architected materials in optimized designs capable of withstanding fracture. Finally, many very recent aspects of fracture resistance, including



**Fig. 19** Unit cell of a material containing randomly distributed square inclusions: (a) initial design and load case 1 geometry; (b) load case 2; (c) load case 3; (d) final design for load cases 1 and 2; (e) final design for load cases 1, 2, and 3. Reproduced from [172] with permission.

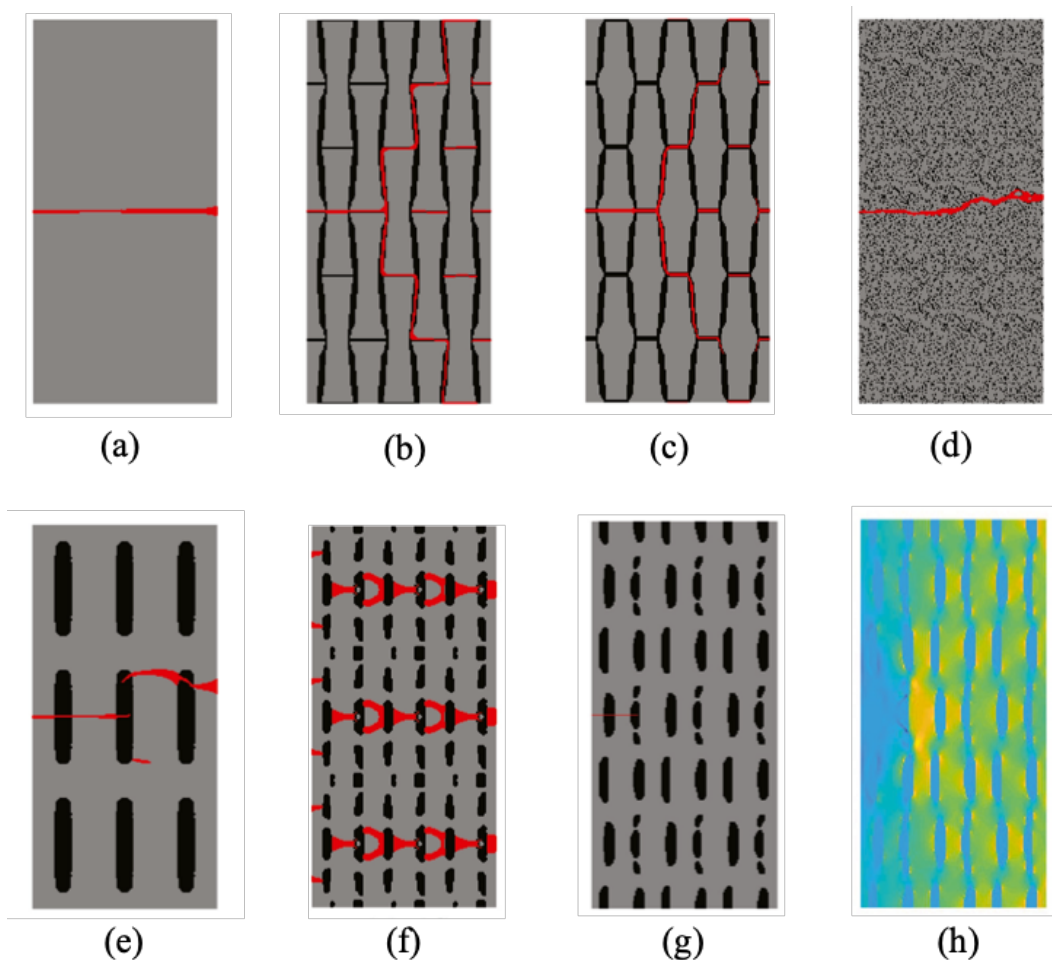
anisotropic effects, dynamic fracture, and composite material design, are still under development and will have to meet the challenges mentioned above, integrating more levels of complexity.

## 8 Conflict of interest statement

On behalf of all authors, the corresponding authors state that there is no conflict of interest.

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**Fig. 20** Comparing final crack patterns of the structure (a-e) with the optimized design structure (f). Additionally, Figures (g) and (h) showcase the topology of an alternative designed structure along with its in-plane stress distribution. Reproduced from [174] with permission.

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