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Sensitivity Analysis of Adjusted Partial Factors for the Assessment of an Existing Reinforced Concrete Bridge

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ABSTRACT: Design standards of Reinforced Concrete (RC) bridges are generally based on a semi-probabilistic format using partial factors to take into account material, geometrical and load effect uncertainties. In the case of an existing bridge, the available information can be used to adjust partial factors given an acceptable target reliability level. However, gathering sufficient information on material properties may not be a straightforward task and the associated assumptions may have an impact on the assessment result. This paper focuses on the assessment of an existing RC bridge at the ultimate limit state using a semi-probabilistic format. First, the data are analysed by evaluating material properties. Then, uncertainty levels are evaluated, and partial factors are adjusted accordingly. Univariate and multivariate analyses are finally conducted to show the main effect of the partial factors' adjustment on the assessment.

Considering the constant ageing of structures, the management of existing bridges is a major topic. To understand the behaviour and needs for repair, assessing structural condition is mandatory. The assessment of an existing RC bridges is usually carried out with a semi-probabilistic approach relying on partial factors determined with the concepts of the reliability theory (Sýkora et al. 2013). If necessary, it is also possible to carry out a full probabilistic approach (Vrouwenvelder 2002). With time, the structural condition and the mechanical prop-

erties of the materials change. Moreover, structures have been designed with varying design codes depending on the conception period, and these design code can drastically change. In Europe, for structures designed before the CEN (2002), load models vary across European countries. The development of industrial process has also changed the level of knowledge and the quality of the materials. Assessment of a structure taking into account its current state requires updated data. However, getting these data and incorporating them into the

assessment require significant in situ campaigns.

Considering the different types of uncertainties (geometrical uncertainties, model uncertainties, variability of the material and statistical uncertainties) is very important when assessing existing structures. Partial factors can be adjusted in accordance with CEN (2002) as described in *fib* Bulletin 80 (2016) where the semi-probabilistic format is adjusted with two types of methods.

The Design Value Method (DVM) is based on principles of EN 1990 (CEN 2002) to calibrate the partial factors used for design. For a particular variable, the partial factor is expressed as a ratio between the First Order Reliability Method (FORM)-based design point coordinate of a variable in the normalised space using standardized sensitivity factors and the characteristic value (or another representative value) of this variable.

A DVM-based approach is used herein, requiring access to adjusted information. Verification for the bending moment is carried out on a girder bridge case study. A sensitivity analysis (relying on the distribution function of the random variables, the coefficient of variation and the target reliability index) has been performed with a univariate approach followed by a multivariate analysis, considering the Yates algorithm as described by Juran and Godfrey (1999).

1. ASSESSMENT OF EXISTING STRUCTURES

Assessing existing structures can be carried out in a multi-level analysis. To validate the assumptions and ensure that they fit the existing structure properly, a maximum of available information should be gathered. Without trying to carry out a full probabilistic approach, partial factors may be adjusted. The design code considered in this analysis is Eurocode 2 (CEN 2005). Each partial factor is adjusted using the DVM presented in *fib* Bulletin 80 (2016) with a 50-year reference period.

1.1. Requirements for performance and safety

The target reliability index is an indicator of the level of requirements expected for a structure in terms of performance and safety. In Europe, this index can be adjusted for existing structures as allowed by XP CEN/TS 17440 (2020). In this study,

the influence of this index is observed in accordance with the recommendations of the *fib* Bulletin 80 (2016). This allows for a comparison of the partial factors with the requirements for new ($\beta = 3.8$) and existing (considering as example $\beta = 3.3$) structures for a traffic loading class 2 with a 50-year reference period.

Adjusting the partial factors implies having knowledge on the associated load or resistance. It is also justified by the fact that for an existing structure lower target reliability level may be used (Steenbergen et al. 2015).

1.2. Analysis of construction regulations and standards

A first structural assessment is generally carried out using the most recent standards as well as considering any material or geometrical data coming from in-situ campaigns. If this first check is not satisfied a more refined study is carried out. The analysis considers the available information through documentation. Apart from the definition of the representative value to be used, parameters may differ according to the design codes, e.g. in France some previous National Standards and guidelines (Circulaire serie A n°8 du 19 Juillet 1934) uses the limiting compressive stress at 90 days for design. Some equations provide ranges of value to switch from the old standards to the new ones (Guide méthodologique Cerema 2016), but this may contribute to introducing additional uncertainties. Similar phenomena are observed in the case of the mechanical properties of steel. When some doubts remain, the most unfavourable value should be used for safety measures. However, this introduces a more important safety margin.

A study of the construction regulations and a knowledge of the history of the materials provide information on the elements. Depending on the design codes, the assessment values vary. For example, the characteristic compressive strength (f_{ck}) is used in CEN (2002) to qualify concrete whereas it was characterized by average compressive strength (f_{cm}) of concrete in France until the 1960s (Circulaire serie A n°8 du 19 Juillet 1934, Poineau et al. 2015b).

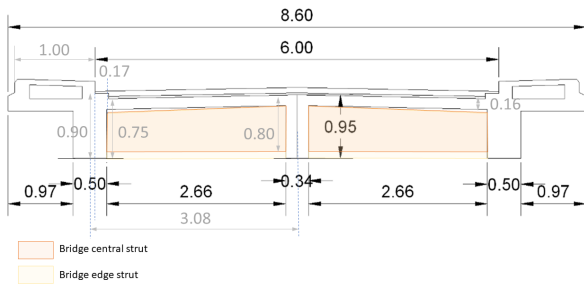


Figure 1: Transversal section of the studied girder bridge

2. APPLICATION TO A GIRDER BRIDGE: EVALUATION OF THE MECHANICAL CHARACTERISTICS

The case study considered herein is a girder bridge built in the late 1950s and supported by three main longitudinal beams (Figure 1). The structural assessment is carried out on the left beam. Longitudinal bending at the ultimate limit state is the primary mode of failure of the structure. No in-situ campaign was carried out on this bridge neither to characterise material properties nor traffic level. Currently, the traffic load on the bridge is limited to 20-ton vehicles.

2.1. Integration of information for the assessment

The random variables and the corresponding partial factors are described in Table 1 where f_c is the compressive strength of concrete, f_y is the yield strength of steel, F_S is the load of the superstructures, ρ_c is the density of concrete, M_{LM1} is the bending moment due to traffic load model 1. Random variables are introduced to account for the uncertainties through partial factors (γ_{Sd} for model uncertainties of load, γ_{Rd1} for model uncertainties of material, γ_{Rd2} for geometrical uncertainties of material and γ_f for statistical uncertainties as presented in Table 1).

2.1.1. Material strength: concrete and steel

To determine the adequate assessment value for materials, details of the initial tests performed during the construction phase are required. However, such data are not systematically available, especially for common bridges. This is the case for

Associated γ	Variable	Distribution	V
γ_c	f_c	Lognormal	10%
γ_s	f_y	Lognormal	5%
γ_{sup}	F_S	Normal	10%
γ_{pp}	ρ_c	Normal	5%
γ_q	M_{LMi}	Gumbel	6.1%
$\gamma_{Rd1,c}$	θf_c	Lognormal	7.5%
$\gamma_{Rd1,s}$	θf_y	Lognormal	5%
$\gamma_{Rd2,c}$	θA_s	Normal	2%
$\gamma_{Rd2,s}$	θh	Normal	2%
γ_{Edsup}	θS	Lognormal	6.5%
γ_{Edpp}	$\theta \rho_c$	Lognormal	7.5%
γ_{Edq}	θM_{LMi}	Lognormal	11%

Table 1: Random variables and associated partial factors γ

this study. Based on the data of 1952, materials with the following mechanical properties are considered : a 250/315 cement dosed at 350 kg/m^3 , a concrete with an admissible compression strength equal to 250 kg/cm^2 at 90 days on a prism measured in tests and a tensile strength of 32.5 kg/cm^2 at 90 days measured by bending of square specimens, and a mild steel working at 1300 kg/cm^2 in tension-compression and 1040 kg/cm^2 in shear (limit of allowable stresses). In the 1950's guaranteed values for design were used. Therefore, mechanical properties considered in the initial design process are most likely guaranteed values.

Concrete According to the Circulaire serie A n°8 du 19 Juillet 1934, with a cement dosage of 20/25 (that is a 200/250 cement) at 350 kg/m^3 , the concrete at 90 days has a minimum guaranteed compressive strength of 235 kg/cm^2 and a minimum guaranteed tensile strength of 32.5 kg/cm^2 .

The methods for testing concrete have changed considerably over time. At the beginning of the reinforced concrete construction period, before the 1960s, the strength was measured on cubic specimens (20 cm x 20 cm) and at 90 days of age. From the 1960s onwards, strength was measured on cylindrical specimens (16 cm x 32 cm) and at 28 days of age. In addition, the considered property value changes from a nominal strength value (20%

of the specimens may not reach the set strength) to a characteristic strength (5% to 10% of the specimens may not reach the set strength) which is the value currently used.

According to the previous French Standards (Titre VI du Fascicule 61 du C.P.C du 29 Décembre 1971), the permissible stress (nominal value) at 28 days is $\sigma_{28} = 180 \text{ bar}$ for a concrete dosed at 250 kg/m^3 . Given the age of the structure, the value of the admissible stress at 90 days can be considered. One uses $\sigma'_j = 1.20 \times \sigma'_{28}$ for concrete classes based on the cement of class 325 and $\sigma'_j = 1.10 \times \sigma'_{28}$ for classes of stronger concrete for j the number of drying days high enough.

In the absence of experimental data, it is possible to use the allowable limits provided in the design codes Circulaire serie A n°8 du 19 Juillet 1934 and Titre VI du Fascicule 61 du C.P.C du 29 Décembre 1971. Guide méthodologique Cerema (2016) reminds a range of value for f_{ck} :

$$0.85f_{cn} < f_{ck} < 0.9f_{cn} \quad (1)$$

where f_{cn} is the nominal compressive strength of concrete at 90 days and f_{ck} is the characteristic compressive strength of concrete at 90 days.

Given the absence of experimental data, $f_{ck} = 18 \text{ MPa}$ on the basis of Titre VI du Fascicule 61 du C.P.C du 29 Décembre 1971 (1968). Thus, the acceptable range of value of the characteristic compressive strength of the concrete is determined with Eq. (1).

Steel According to Poineau et al. (2015c) and Poineau et al. (2015a), the mechanical properties and the methods of characterisation and control of steel reinforcement for RC structures have considerably evolved since the end of the 19th century. Until the 1960s, steel reinforcement had a yield strength higher than or equal to a guaranteed value f_{eg} .

According to Poineau et al. (2015c), f_{eg} is determined from a large batch of tested samples (less than 0.25% are accepted under the guaranteed value).

$$f_{eg} = f_{em} - 2s \quad (2)$$

Era	Type of steel	Limit state for calculation	Permissible limit	Guaranteed value
(i)	Smooth round mild steels	Yield strength	24 kgf/mm ²	
(i)	Smooth round mild steels	Breaking strength	42 kgf/mm ²	
(ii)	Smooth round mild steels	Yield strength	1300 kg/cm ²	
(ii)	Smooth round mild steels	Extension or compression of steel	13 kg (i.e. 130 MPa)	
(ii)	Smooth round mild steels	Yield point	yield point = 2 × fatigue limit if fracture limit < $\frac{1}{3}$ fatigue limit	
(iii)	Smooth round mild steels	Yield strength		24 kgf/mm ² to 36 kgf/mm ² (i.e. 235 to 350 MPa)
From the 1950s onwards	High bond reinforcement	Yield strength		40 kgf/mm ² to 42 kgf/mm ² (i.e. 390 to 410 MPa)
(iv)	Mild steels	Stresses	1600 kg/cm ²	
(iv)	Mild steels	Yield strength in tension	Nominal yield strength in tension, assumed to be equal to that in compression	
(iv)	Fe E 22 steel grades	Fatigue limit	22 kgf/mm ² (i.e. 215 MPa)	> 40 kgf/mm ² (or 390 MPa)

Table 2: Mechanical properties of steel rebar reported in past French Standards

Nota : (i) Charges générales du 29 Octobre 1913 (1913), (ii) Circulaire série A n°3 du 10 Mai 1927 (1927), (iii) Poineau et al. (2015c), (iv) Titre VI du Fascicule 61 du C.P.C du 29 Décembre 1971 (1968).

where f_{em} is the mean value and s is the standard deviation of yield strength of five reinforcement steel samples.

A batch of reinforcement was considered to be compliant if, after considering five samples, it had an elastic limit greater than f_{eg} (less than 0.5% of tests are under the guaranteed value). Otherwise, five more samples were tested hence providing test results for a batch of 10 samples. In both case the quantile is given in Eq. (3).

$$f_{em1} - 1.8 \times s_1 \geq f_{eg} \quad (3)$$

where f_{em1} is the mean value and s_1 is the standard deviation of yield strength of the tested reinforcement steel from the testes batch. It is noted that a normal distribution is considered here.

Eqs. (2) and (3) show that the characteristic value for current design codes (5% fractile) are superior to the guaranteed value (2.5% fractile). However, it was only in the 1980's that proper statistical methods were used. For old structures, these changes

make it more difficult to determine the value of f_{yk} directly based on the steel reinforcement used. According to the design codes, different limits are used, referring to multiple notions of fatigue limit, elasticity limit, or rupture limit.

In the absence of experimental data, the only information about steel properties for this case study come from the design calculation of 1952. However, no indication is given on the steel grade.

Table 2 shows the different regulations that have been used to design and to assess bridges designed in the 1950's. The properties according to these standards significantly vary and several assumptions have to be considered. It is likely that the type of steel used in the studied bridge were plain bars given the construction period. For comparison, a study carried out in Italy by Verderame et al. (1950) states that AQ42 steel grade were the most commonly used in the early 1960's. The mean yield strength of AQ42 steels from a batch of 3520 specimens is 322 MPa (which is a higher value than considered for case S1 and S3 and the minimum value is 210 MPa which lower than the guaranteed value considered for this case study).

The coefficient of variation for yield strength is an important when adjusting the partial factor for steel. According to JCSS (2001), it ranges from 0.02 to 0.03.

Load The calculation of the bending moment is carried out according to a loading model LM1 and a loading model LM3 with a special vehicle limited to 20 tons. Considering the CEN (2002) loading model and the 20-ton vehicles restriction give two load cases for traffic. The beam analysis is performed using the software ST1 Cerema ITM (2022).

In summary, the determination of the yield strength, concrete compressive strength and load bending moment are complex since little information are available. Therefore, a parametric study is carried out and the reliability of the bridge is assessed by considering the lowest value for resistance parameters and the highest one for load parameters. If the structure is not reliable, then the

Subcase	S1	S2	S3	S4
f_{ck} (MPa)	18	18	18	18
f_{yk} (MPa)	235	395	235	395
M_Q (MN.m)	0.776	0.776	0.559	0.559

Table 3: Details of subcases S1, S2, S3 and S4

verification can be carried out with higher values for material strength properties after a test campaign. Combination of these values provide several subcases. For the four subcases S1, S2, S3 and S4 presented in Table 3, three sets of partial factors are tested; partial factors provided by CEN (2002), adjusted partial factors with a target reliability index $\beta_t = 3.8$ and adjusted partial factors with a lower target reliability index $\beta_t = 3.3$.

It is noted that f_{ck} was fixed in these four subcases as in this study, detailed analysis of data and design codes provide a small range of values to describe the compressive strength of concrete.

The verification of the weight bearing capacity of the structure is assessed with Eq. (4):

$$\mathcal{G} = M_{Ra} - M_{Sa} \quad (4)$$

where \mathcal{G} is the margin function, M_{Ra} the calculated resisting bending moment value for a RC beam, and M_{Sa} the load bending moment value within a semi-probabilistic format.

If the bending moment due to load is lower than the moment of resistance ($\mathcal{G} \geq 0$ in Eq. (4)), the beam is considered reliable according to the chosen reliability target. In the case S4, the structure is reliable with the partial factors recommended by CEN (2002). Therefore, adjusted partial factors are not required and the subcase S4 is not considered in the following.

3. RESULTS AND DISCUSSION

The adjusted partial factors are calculated for the variables specified in Table 1. The impact of the assumptions made on the different variables is assessed. The sensitivity analysis considers the coefficient of variation, the type of variables, and the target reliability index. First a univariate sensitivity analysis on the partial factors is carried out. Then

Subcase	S1	S2	S3	S4
EN	-0.921	-0.512	-0.632	0.536
DVM: $\beta_0 = 3.8$	-0.649	-0.156	-0.388	0.775
DVM: $\beta_1 = 3.3$	-0.566	-0.059	-0.319	0.824

Table 4: Calculated \mathcal{G} value for S1, S2, S3 and S4 subcases (in MN.m)

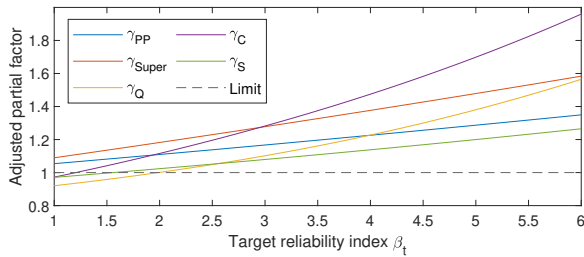


Figure 2: Influence of the target reliability index on partial factors

the analysis is carried out on the assessment result with a univariate and then multivariate sensitivity analysis.

The margin function \mathcal{G} presented in Eq. (4) is obtained from the adjustment with the data given in Table 1. The values of \mathcal{G} are presented in Table 4. At the ultimate limit state, even with the parametric approach, the bridge does not satisfy the safety requirements, except for subcase S4.

3.1. Univariate sensitivity analysis on the partial factors

Target reliability index The higher the target, the larger the partial factor as shown in Figure 2. In practice when assessing an existing structure, partial factors are not adjusted if the updated partial factor is larger than the value recommended for design.

For the concrete and traffic, the partial factors increase faster than for the other parameters.

Distribution type and coefficient of variation

Partial factors are adjusted based on random variables (that represent uncertainties). In the absence of experimental data collected on the bridge, literature is used to update the associated probability function and in particular according to the recommendations of JCSS (2001). Depending on the dis-

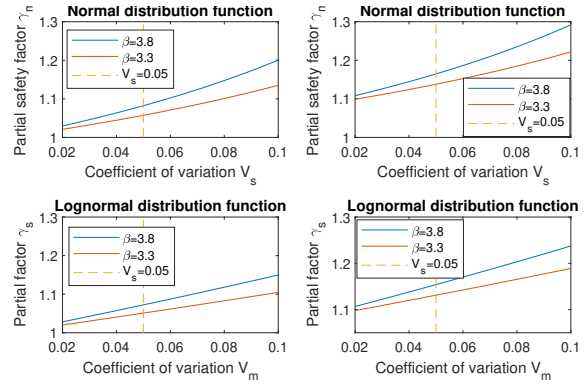


Figure 3: Influence of the distribution type for steel and concrete

tribution function, the formulas for adjustment is different (fib Bulletin 80 2016). These coefficients of variation also depend on the quality control practice of the materials, the traffic variations, and the other climatic actions. Figure 3 shows, as expected, that the higher the coefficient of variation is, the higher the partial factor is, hence is importance of having reliable information.

Figure 3 also illustrates the effect of the distribution type (normal and lognormal) and of the coefficient of variation on the partial for steel and concrete. Partial factors are higher when the distribution type is a normal function and as expected they increase as the coefficients of variation increase.

3.2. Univariate sensitivity analyses of the assessment result

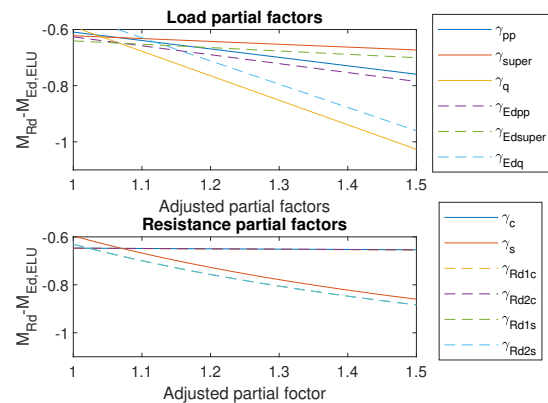


Figure 4: Influence of the adjustment of partial factor

Overall results of the assessment are very sensitive to adjustment. Table 4 gives the general result

of the assessment with the calculated \mathcal{G} considering several cases and indicates that the impact of the adjustment on the assessment is greater if partial factors decrease. The comparison of the assessment results calculated with the Eurocode 2 CEN (2005) and those calculated with adjusted partial factors, implies that the impact of using adjusted factors is significant. Figure 4 shows that the most important partial factors are: γ_q , γ_{Edq} , γ_{pp} , γ_{Edpp} , γ_s and γ_{Rd2s} . This observation is used for the multivariate sensitivity analysis.

3.3. Multivariate sensitivity analysis

The correlation between the variables and their effect on the margin function \mathcal{G} is analysed with a multivariate sensitivity analysis method based on the Yates algorithm (Juran and Godfrey 1999) to perform a factorial analysis. In factorial analysis, the effect are described as "main effects" and "interaction effects". The Yates algorithm allows to separate estimation of the individual effects and the interaction effects of the factors.

To perform a Yates analysis, 2^k observations are first defined using lower (-) and upper (+) values (called levels) picked for the selected k parameters $x_i, i \in [1..k]$. The individual and combined effects δ_i and $\delta_{i,j}$ of the parameters are then identified to define a multilinear regression model (Hunter 1966):

$$\mathcal{G} = \mathcal{G}_0 + \sum_{i=1}^k \delta_i x_i + \sum_{i=1}^k \sum_{j=i+1}^k \delta_{i,j} x_i x_j + \sum_{i=1}^k \sum_{j=i+1}^k \sum_{l=j+1}^k \delta_{i,j,l} x_i x_j x_l + \dots \quad (5)$$

Estimated effects were determined for $\beta_t = 3.8$ and $\beta_t = 3.3$.

Factors	Letter	Level (-)	Level (+)
V_{pp}	A	0.01	0.1
μ_q	B	0.559	0.776
σ_q	C	0.028	0.116
V_s	D	0.02	0.15
θ_{pp}	E	0.01	0.1
θ_q	F	0.06	0.11
θ_s	G	0.02	0.1

Table 5: Values used for Yates algorithm

Observations are named with lower case letters. A letter appears in the name of the observation

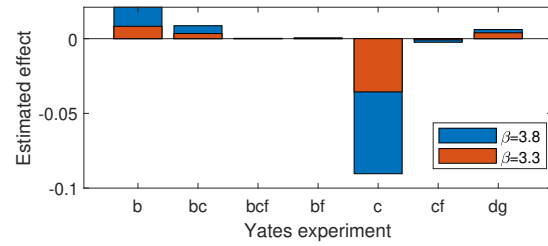


Figure 5: Estimated effects of the multivariate sensitivity analysis (Yates algorithm)

when the factor (called with an upper-case letter) is used at the upper value: Level (+). For example, the observation ae is the case where only the variables A and E are at Level (+) whereas the other variables are at Level (-).

Estimated effects are observed in (Figure 5) for the following observations: $a, ae, b, bc, bcf, bf, c, cf, d, dg, e, f, g$. Only observations with a non-zero estimated effect are represented in Figure 5. The higher the estimated effects are, the higher the impacts on the assessment result (measured by \mathcal{G}) are. Cases with two letters indicate the correlated effects. The partial factors do not interact significantly with each other which was expected considering that the limit state is linear. The largest estimated effects are observed for the cases where the variation of a single coefficient is considered. For material, the most important partial factor is the one on steel (observation dg) and for load it the partial factor for traffic.

In addition, the signs of the individual effects are consistent. Indeed, the required margin function \mathcal{G} is expected to increase with the mean of traffic-related variable μ_q , but logically decreases with its standard deviation σ_q . The combined effect between these two variables is small: it is consistent with the Table C3 of Eurocode 0 (CEN 2002) where the design value defined for a traffic Gumbel distribution is a linear combination of its mean and standard deviation. Finally, the magnitude of effects on the required margin function \mathcal{G} increases with the required reliability.

4. CONCLUSION

This study focuses on the adjustment of partial factors for a girder bridge case study. An analysis of

the beam has been carried out with EN and partial factors adjusted with DVM. The margin function \mathcal{G} is a linear function based on the verification of bending moment.

Uncertainties have been evaluated from previous case studies and information obtained in the literature. A sensitivity analysis is proposed so as to unveil the effects of scalar parameters describing the loading and behaviours of steel and concrete as random variables on the required margin function. The load induced by traffic (for load) and the yield strength of steel (for material) prove to be the most dominant parameters for the considered RC beam. These parameters have to be properly determined in order to have precise assessment. Furthermore, the Yates analysis shows that uncertainties on parameters do not significantly interfere with one another.

Converting past knowledge and prescriptions so as to make a meaningful use of current standards to maintain existing structures remains a tedious task.

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